EVALUATION OF MATHEMATICAL MODELS AC MAGNETIC CIRCUITS

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ABSTRACT

Present paper shows that even seemingly acceptable linearization of the mathematical model of harmonically excited nonlinear magnetic circuit may lead to appreciable errors.

Keywords: *magnetic circuits, numerical methods, mathematical models, validity of models, finite element analysis.*

1. INTRODUCTION

When a mathematical model of AC magnetic circuits of electrical machines and devices is built, its linearization is often done, i.e. the course of permeability $\mu = \mu(H)$ of the magnetic circuit is replaced by a constant value. The error caused by the linearization is intuitively considered as negligible. The solution of a mathematical model with linearization of permeability is much simpler, because harmonically complex representation of the time-variable waveforms is possible [1], [2]. Presented paper shows that primarily when magnetic heavily saturated magnetic circuits are modelled (mentioned devices are normally working under this condition), the linearization of magnetic properties may lead to totally inaccurate results. This fact will be demonstrated on a simple configuration of an alternately magnetized ferromagnetic cylinder. Admissibility of the linearization of the magnetization curve of the cylinder is decided by the comparison of the Joule losses caused by eddy currents induced in the cylinder.

2. FORMULATION OF THE PROBLEM

To ensure good accuracy and reliability of the solution of an AC magnetic circuit, it is necessary to deal with the validity of the used mathematical model. We focus on a simple magnetic circuit according to Fig.1, which consists of a ferrite jacket ($\mu_r = 10^4$, $\gamma \to 0$), solid cylinder of electrical steel (its magnetic curve is shown in Fig. 2, $\gamma = 1{,}4 \,.\, 10^7$ S / m) and a coil of a thin wire made of copper (skin effect is negligible) with the current density $J(t) = J_0 \sin \omega t$ ($\omega = 2\pi f$). The magnetic field in the cylinder is calculated using both nonlinear model and by using the linearized model. The results of both calculations are compared based on the values of Joule losses due to eddy currents in the cylinder. The derivation of the following equations is described in [1]. Numerical solutions are carried out using the Quick Field 5.0 [3].

2.1. Nonlinear mathematical model

<u>Equations</u>. The equation for the magnetic vector potential $\boldsymbol{A}$ is

$$\mathrm{rot}\frac{1}{\mu}\mathrm{rot}\,\boldsymbol{A} = -\gamma\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{J} \quad (1)$$

For 2D cylindrical coordinates is $\boldsymbol{A} = \boldsymbol{\alpha}_0 A_\alpha(r, z, t)$, $\boldsymbol{J} = \boldsymbol{\alpha}_0 J_\alpha$. Eq. (1) passes to

$$\frac{\partial}{\partial r}\left[\frac{1}{\mu r}\frac{\partial}{\partial r}\left(r A_\alpha\right)\right] + \frac{\partial}{\partial z}\left[\frac{1}{\mu}\frac{\partial}{\partial z}\left(r A_\alpha\right)\right] = \gamma\frac{\partial A_\alpha}{\partial t} - J_\alpha \quad (2)$$

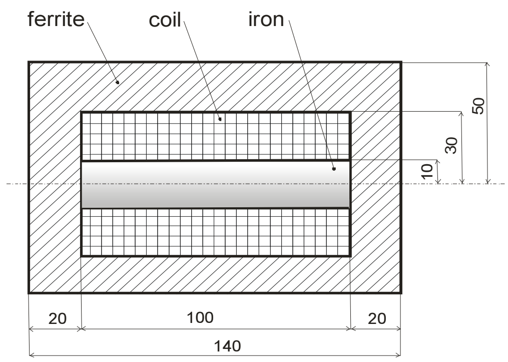

Fig. 1 Nonlinear AC magnetic circuit

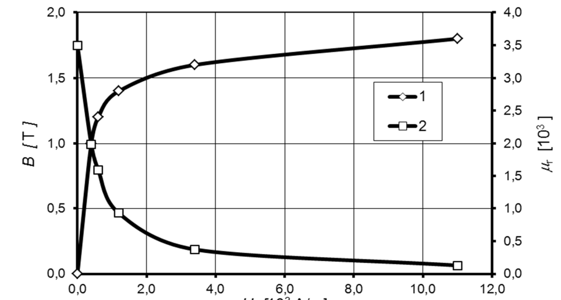

Fig. 2 Magnetization curve of the electrical steel; 1….. $B = B(H)$, 2….. $\mu_r = \mu_r(H)$

<u>The definition area</u>. Because of the symmetry we work with one quarter of the area in which the magnetic field is distributed (Fig. 3): domain Ω is composed of four subareas: $\Omega = \Omega_1 + \Omega_2 + \Omega_3 + \Omega_4$ wherein subarea Ω_1 corresponds to the ferrite coat, subarea Ω_2 corresponds to a cross-section of the coil, subarea Ω_3 corresponds to a ferromagnetic cylinder and subarea Ω_4 corresponds to the surrounding air.

Based on the knowledge of the field of $A_\alpha(r, z, t)$ can be determined:

• vector components of the magnetic flux density **B**:

$$B_r\,(r,z,t) = -\frac{\partial A_\alpha}{\partial z},\;\; B_z(r,z,t) = \frac{\partial A_\alpha}{\partial r} + \frac{A_\alpha}{r} \qquad (3)$$

• current density of eddy currents induced in the cylinder by time-varying magnetic field

$$J_{\text{eddy},\alpha}(r,z,t) = \gamma \frac{\partial A_z}{\partial t} \qquad (4)$$

• Joule losses density caused by eddy currents

$$w_\text{J}(r,z,t) = \frac{J^2_{\text{eddy},\alpha}}{\gamma} \qquad (5)$$

Boundary conditions, Fig. 3; (t > 0):

$\overline{\text{AB}}$: $A_\alpha = 0$ .... antisymmety to the asis r = 0

$\overline{\text{BC}}, \overline{\text{CD}}$: $A_\alpha = 0$ .... fictive boundary Γ_∞ continuity of the magnetic field (6)

$\overline{\text{DA}}$: $\frac{\partial A_\alpha}{\partial z} = 0$ ..... symmetry to the plane z = 0

Initial condition $(t = 0)$:

$\Omega_1, ..., \Omega_4$: $A_\alpha = 0$ ..... the coil is not powered (7)

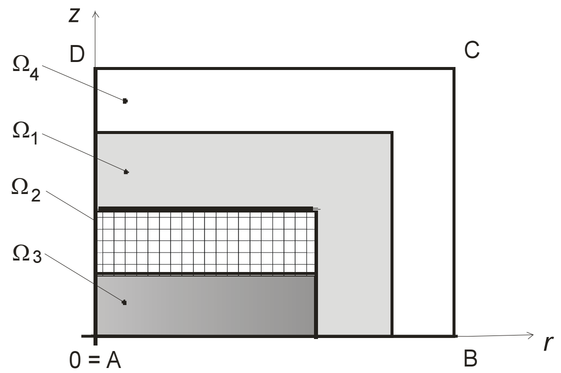

Fig. 3 Definition domain Ω and its subdomains $\Omega_1, ..., \Omega_4$.

2.2. The linearized mathematical model

For the time-varying magnetic field in the linearized surroundings is valid

$$\text{rot rot}\,\boldsymbol{A} = -\gamma\,\mu \frac{\partial \boldsymbol{A}}{\partial t} + \mu\,\boldsymbol{J} \qquad (8)$$

The Helmholtz equation in the phasor expression is

$$\text{rot rot}\,\underline{\boldsymbol{A}} + \mathbf{k}^2 \underline{\boldsymbol{A}} = \mu \underline{\boldsymbol{J}},\; \mathbf{k} = -\text{j}\omega\mu\gamma,\; \omega = 2\pi f \qquad (9)$$

where $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{J}}$ are phasors of vectors $\boldsymbol{A}$, $\boldsymbol{J}$ and μ_s is usually set as the mean permeability $\mu = \mu(H)$ on the interval < 0, H_0 > where the intensity of magnetic field H_0 corresponds to the amplitude of the excitation current I_0:

$$\mu_\text{midd} = \frac{1}{H_0}\int_0^{H_0} \mu(H)\,\text{d}H \qquad (10)$$

For 2D, the magnetic field in cylindrical coordinates (r, z) shall take form of the Eq. (8)

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\,A_\alpha)\right] + \frac{\partial^2 A_\alpha}{\partial z^2} = \mu_\text{midd}\,\gamma \frac{\partial A_\alpha}{\partial t} - \mu_\text{midd} J_\alpha \qquad (11)$$

and Eq. (9) passes to

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\,\underline{A}_\alpha)\right] + \frac{\partial^2 A_\alpha}{\partial z^2} + \mathbf{k}^2 \underline{A}_\alpha = -\mu_\text{midd} \underline{J}_\alpha \qquad (12)$$

where $\underline{A}_\alpha = \underline{A}_\alpha(r,z)$ and $\underline{J}_\alpha = \text{const.}$ are phasors. For quantities $J_{\text{eddy},\alpha}$ and $w_\text{J} = \frac{1}{\gamma} J^2_{\text{eddy},\alpha}$ is valid

$$\underline{J}_{\text{eddy},\alpha}(r,z) = -\text{j}\,\omega\,\gamma\,\mu_\text{midd}\,\underline{A}_\alpha \qquad (13)$$

$$w_\text{J} = w_\text{J}(r,z) = \frac{1}{\gamma}\left(\underline{J}_{\text{eddy},\alpha} \cdot \underline{J}^*_{\text{eddy},\alpha}\right) \qquad (14)$$

where $\underline{J}^*_{\text{eddy},\alpha}$ is the complex conjugated phasor to the phasor. Usually with an effective complex value is worked

$$\underline{J}_{\text{eddy},\alpha,\text{ef}} = \underline{J}_{\text{eddy},\alpha} \big/ \sqrt{2} \qquad (15)$$

and hence

$$w_\text{J}(r,z) = \frac{1}{2\gamma}\left(\underline{J}_{\text{eddy},\alpha,\text{ef}} \cdot \underline{J}^*_{\text{eddy},\alpha,\text{ef}}\right) \qquad (16)$$

3. COMPUTER MODEL

Numerical solutions of these mathematical models were performed with the use of FEM program QuickField 5.0. Nonlinear model was solved under module *Transient Electromagnetics* and linearized model under module *Transient Electromagnetics* and also under module *Time-Harmonic Magnetics* (in phasor formulation). Fig. 4 presents a sample of the calculated course of the lines of the magnetic field $A_\alpha(r,z,t) = \text{const.}$ For the nonlinear model the convergence of numerical solutions was studied, i.e. the influence of the discretization at the geometric discretization range δ_min, δ_max (Tab.1) was examined, as well as the influence of the discretization of time Δt (Tab. 2). Tab.1 shows that the accuracy of the results is influenced by values δ_min, i.e. the discretization of the subarea Ω_1 (cylinder), but the influence of the discretization δ_max of the subarea Ω_4 (environment) is negligible considering the accuracy of the magnetic field in the cylinder. Comparing both Tables 1 and 2 shows that the effect of the discretization Δt compared to the influence δ_min is not signifi- cant. Thus, the values set by the space-time discretization therefore provide acceptable accuracy of the solution.

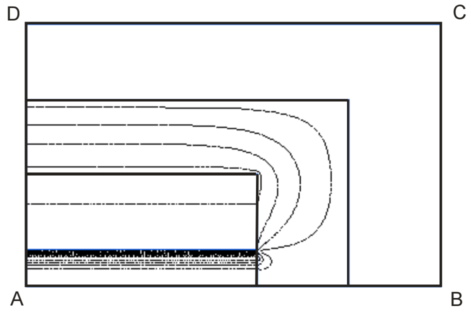

Fig. 4 The course magnetic field lines of force according tothe nonlinear mathematical model; $J_\alpha = 1.10^6$ A/m^2 , f = 50Hz, $t/T_\mathrm{p} = 0.75$, scale of magnetic field lines: 4.10^{-5} Wb

Table 1 The convergence of the numerical solution: the influence of the smoothness of the used geometrical mesh; (first period, $J_\alpha = 2.10^6$ A/m^2 , f = 50 Hz)

	w_J [10^6 W/m^3]			
	1,0E-3	1,0E-4	2,5E-4	δ_{min} [m]
	1,0E-3^3			δ_{max} [m]
t / T_p	1,0E-4			Δt [sec]
0,1	4,893	4,872	4,876	
0,2	6,338	6,413	6,357	
0,3	4,215	4,176	4,181	
0,4	1,337	1,296	1,288	
0,5	0,595	0,573	0,578	
0,6	6,909	7,379	7,339	
0,7	9,286	9,437	9,433	
0,8	6,324	6,391	6,389	
0,9	1,913	1,987	1,957	
1,0	0,422	0,411	0,416	

Table 2 The convergence of the numerical solution: the Influence of the of the used space-time mesh (first period, $J_\alpha = 2.10^6$ Am^{-2}, f = 50 Hz)

	w_J [10^6 W/m^3]			
	1,0E-3	5,0E-4	2,5E-4	δ_{min} [m]
	1,0E-3^3			δ_{max} [m]
t /T_p	1,0E-4	5,0E-5	2,5E-5^5	Δt [sec]
0,1	4,893	4,898	4,900	
0,2	6,338	6,443	6,380	
0,3	4,215	4,184	4,195	
0,4	1,337	1,301	1,295	
0,5	0,595	0,585	0,600	
0,6	6,909	7,400	7,390	
0,7	9,286	9,456	9,453	
0,8	6,324	6,397	6,334	
0,9	1,913	1,954	1,960	
1,0	0,422	0,428	0,437	

4. VERIFICATION CALCULATIONS

4.1. Input data of the verification calculations

The calculation was made for these current densities: $J = J_0 \sin \omega t$, where $J_0 = J1 = 1.10^6$, $J_0 = J2 = 2.10^6$ and $J_0 = J3 = 3.10^6$ Am^{-2}. The mean value of the relative permeability of the electrical steel is counted from its magnetization characteristics (Fig. 2): $\mu_{\mathrm{r,midd}} \approx 505$. Other physical parameters are in paragraph 2.

4.2. Results of the verification calculations and their discussion

The time needed for the solution of the magnetic field of the AC powered nonlinear model to reach the steady state was observed. From the Tab. 3 it is clear that the steady state is reached relatively quickly, the differences between the fifth and the tenth period are negligible. Tenth period can therefore be declared as the steady state solution for the nonlinear model and therefore also for the linearized model. The following results stand for the tenth period of the AC.

Table 3 Steadying of the time variant magnetic field excited by the harmonic current; solution of the nonlinear mathematical model, $J_\alpha = 2.10^6$ A/m^2 , f = 50 Hz

	w_J [10^6 W/m^3]		
t/T_p	1. perioda	5. perioda	10. perioda
0,0	0,000	0,420	0,421
0,1	4,893	6,853	6,854
0,2	6,338	9,122	9,130
0,3	4,215	6,017	6,036
0,4	1,337	1,829	1,834
0,5	0,595	0,420	0,420
0,6	6,909	6,857	6,856
0,7	9,286	9,151	9,146
0,8	6,324	6,077	6,065
0,9	1,913	1,842	1,839
1,0	0,422	0,421	0,421

Joule losses density w_J in the cylinder was calculated both for the nonlinear model and for the linearized model at different current densities J_α (Fig. 5 and Fig. 6). The mean values at time levels t were calculated as:

$$W_{\mathrm{J,s}}(t) = \frac{1}{V_1} \int_{V_1} w_\mathrm{J}(r,z,t)\,\mathrm{d}V \qquad (17)$$

wherein V_1 is the volume of the cylinder. It was found that there is a significant difference between the two solutions; *the losses in the nonlinear model are significantly lower,*

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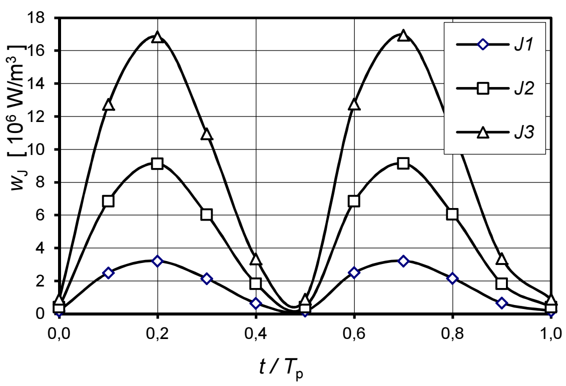

Fig. 5 The influence of J on $w_{J,n}\left[\mathrm{W/m^3}\right]$ for the nonlinear transient problem; tenth period, $f = 50\ \mathrm{Hz} \approx T_p = 0.02\ \mathrm{s}$, **1…**$J1 = 1.10^6 \mathrm{Am^{-2}}$, **2…** $J2 = 2.10^6 \mathrm{Am^{-2}}$, **3…** $J3 = 3.10^6\ \mathrm{Am^{-2}}$

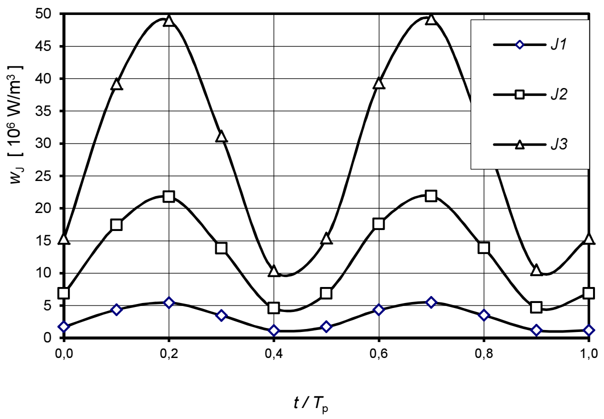

Fig. 6 The influence of J_α on $w_{J,\mathrm{midd}}\left[\mathrm{W/m^3}\right]$ for the linearized transient problem; tenth period, $f = 50\ \mathrm{Hz} \approx T_p = 0.02\ \mathrm{s}$, $\mu_{\mathrm{r\,midd}} = 505$, **1…**$J1 = 1.10^6\ \mathrm{Am^{-2}}$, **2…** $J2 = 2.10^6 \mathrm{Am^{-2}}$, **3…** $J3 = 3.10^6\ \mathrm{Am^{-2}}$

and the difference increases with the excitation current. Linearized solution obviously includes a big error.

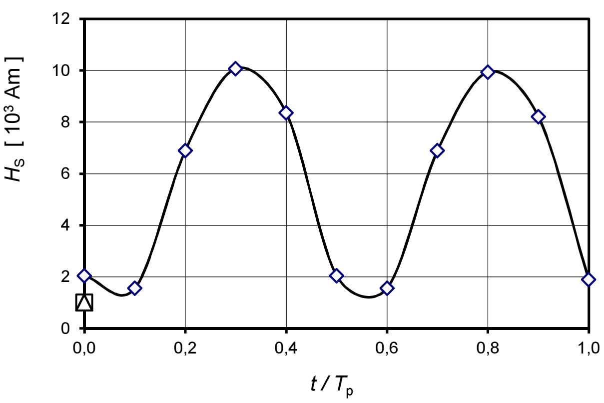

Fig. 7 The time function of the mean value of the magnetic field intensity H_S in the nonlinear cylinder (tenth period, f = 50 Hz, $T_p = 0.02\ \mathrm{s}$, $J_\alpha = 2$ [10^6 Am^{-2}]

This finding is in accordance with the following considerations. Let us determine the steady time course of the mean value of the intensity of the magnetic field $H_{\mathrm{midd}}(t)$ in the cylinder (Fig. 7). Then, with regard to the course of $\mu_r(H)$ shown in Fig. 2, it is apparent that the values μ_r in the nonlinear model are generally less than the value $\mu_{\mathrm{r,midd}} \approx 505$, which we used in the linearized model. This difference increases with the value of J_α. It is obvious that the values W_J and $J_{\mathrm{eddy},\alpha}$ are lower for the nonlinear model than the corresponding values for the linearized model.

5. CONCLUSION

The effect of linearization of the mathematical model of the ferromagnetic cylinder in the alternating magnetic field was evaluated in this paper. Although linearization allows to calculate with the use of phasors, which significantly simplifies the numerical solution, it can cause a large error in results. It is therefore recommended to give proper attention to the question of the eligibility of the linearization of the mathematical model and in case of doubt (e.g. for larger values of the flux density), to give priority to the computationally more laborious, but more reliable nonlinear mathematical model.

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BIOGRAPHIES

Daniel Mayer was born in 1930. Received the Ing., PhD. and DrSc. degrees in electrical engineering from Technical University in Prague, Czech Republic, in 1952, 1958 and 1979, respectively. In 1959 Associate Professor at the University of West Bohemia in Pilsen, in 1968 Full Professor of the Theory of Electrical Engineering. Research interests: circuit theory, electromagnetic field theory, electrical machines and apparatus, history of electrical engineering. He published 6 books, more than 270 scientific papers and 11 patents. He is a Fellow of the IET, member ICS and member of editorial boards of several international journals.

Bohus Ulrych Assoc. Prof. was born in 1937, has been working for a long time in the Department of Theory of Electrical Engineering at the Faculty of Electrical Engineering of UWB in Plzeň. His professional interests are aimed at modern numerical methods of solution of electromagnetic and coupled problems. Author and co-author of about 160 papers and several textbooks. Author of a lot of user's SW for calculation of electromagnetic fields and coupled problems.