

MATHEMATICAL MODELING OF OPTIONS USING GEOGEBRA

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ABSTRACT

The aim of this contribution is to analyze the application of informatics software package GeoGebra in the modeling of options strategy. Several specific examples are presented. The object of this study is behavior of overall profit in different options strategies, observed from graphic point of view. Based on the analyzed examples using GeoGebra slider feature we conclude that bull spread and bear spread both provide limited profit and loss. In addition, for specific values of parameters profit is strictly positive.

Keywords: options, bull spread, bear spread, GeoGebra

1. INTRODUCTION

GeoGebra is open-source software for mathematics teaching and learning that offers geometry, algebra and calculus features in a fully connected and easy-to-use software environment. It is available free of charge and used by thousands of students and teachers around the world in classrooms and at home. Unlike with commercial products, GeoGebra is not designed to be used only by schools or universities allowed by site licenses, but by private users as well. The relationship between functions and graphics found its application in the field of financial mathematics, thus it is widely used to explain financial instruments on a market such as options [1, 2].

2. SUBJECT

An **option** is a contract that gives the buyer the right, but not the obligation, to buy or sell an underlying asset at a specific price on or before a certain date. An option, just like a stock or bond, is a security. It is also a binding contract with strictly defined terms and properties. There are two types of options, calls and puts:

A **call** gives the holder the right to buy an asset at a certain price within a specific period of time. Calls are similar to having a long position on a stock. Buyers of calls hope that the price of the stock will increase substantially before the option expires.

A **put** gives the holder the right to sell an asset at a certain price within a specific period of time. Puts are very similar to having a short position on a stock. Buyers of puts hope that the price of the stock will fall before the option expires.

The price at which an underlying stock can be purchased or sold is called the strike price. This is the price a stock price must go above (for calls) or go below (for puts) before a position can be exercised for a profit. All of this must occur before the expiration date. The total cost (the price) of an option is called the premium.

Analyzing the relationship between call and put prices opens up a range of complex options strategies. Amongst them are bull spread and bear spread, which will be analyzed in this contribution.

Bull spread is a bullish, vertical spread options strategy that is designed to profit from a moderate rise in the price

of the underlying security. A bull spread consists of two or more options of the same type, and results in profit when the underlying asset increases. This strategy can be constructed using either put options or call options, hence we have bull call spread and bear put spread.

A bull call spread is constructed by buying a call option with a low exercise price, and selling another call option with a higher exercise price. A bull put spread is constructed by selling higher striking put options and buying the same number of lower striking put options on the same underlying security with the same expiration date.

Bear spread is a bearish options strategy which is used when the options trader is moderately pessimistic about the value of underlying asset. A bear spread consists of two or more options of the same type, and results in profit when the underlying asset decreases (the opposite to bull spread). This strategy can be constructed by either using call or put options and therefore we have bear call spread and bear put spread.

Bear call/put spread is entered by buying call/put option of a higher strike price and writing the same number of call/put options of a lower strike price on the same asset and same expiration month [3–6].

3. METHODS

Using GeoGebra, a couple of bull spread and bear spread examples have been analyzed. We used GeoGebra to see how the total profit behaves depending on different parameters.

In order to analyze behavior of overall profit function depending on premiums and strike prices we created four sliders: B , S , B_p , S_p , where B and S stand for strike price for bought and sold option respectively, B_p and S_p stand for their premium respectively. All four sliders are strictly positive, according to their economic meaning. GeoGebra lets us use sliders when creating function, which means that function is dependent on values of sliders. This GeoGebra feature is suitable for observing behavior of various functions depending on values of parameters. Having this in mind we formed Buy and Sell functions, which represent profit from buying and selling option respectively, as:

$$\text{Buy}(St) = \begin{cases} -B_p & \text{if } St < B, \\ St - B - B_p & \text{if } St \geq B, \end{cases}$$

$$\text{Sell}(St) = \begin{cases} S_p & \text{if } St < S, \\ S + S_p - St & \text{if } St \geq S. \end{cases}$$

Overall profit function is result of these two:
 $\text{Profit}(St) = \text{Buy}(St) + \text{Sell}(St)$.

GeoGebra allows us to change slider value directly in the graph window and see the change in graph immediately. This GeoGebra property is especially convenient to see what happens with profit for specific values of sliders and vice versa what values of sliders are needed for specific

behavior of profit.

4. RESULTS

Example 4.1. A trader might construct a bull spread by buying a \$3 call with a \$30 strike price and selling a \$1 call with a \$35 strike. How does the profit change with increase of premium of buying option with remaining parameters staying the same?

Table 1 Analyzing profit/loss for bull call spread

St (Actual price)	Profit/Loss	Explanation
$St < 30$	-2	Neither option is exercised, therefore traders profit is $1 - 3 = -2$
$30 < St < 35$	$St - 32$	Only \$30 option is exercised $1 - 3 + (St - 30) = St - 32$
$35 < St$	3	Both options are exercised, so overall profit is $1 - 3 - (St - 35) + (St - 30) = 3$.

Table 2 Analyzing profit/loss for bear call spread

St (Actual price)	Profit/Loss	Explanation
$St < 30$	2	Neither option is exercised, therefore traders profit is $4 - 2 = 2$
$30 < St < 35$	$32 - St$	Only \$30 option is exercised, but since trader has sold it profit is $4 - 2 - (St - 30) = 32 - St$
$35 < St$	-3	Both options are exercised resulting in traders loss $4 - 2 - (St - 30) + (St - 35) = -3$.

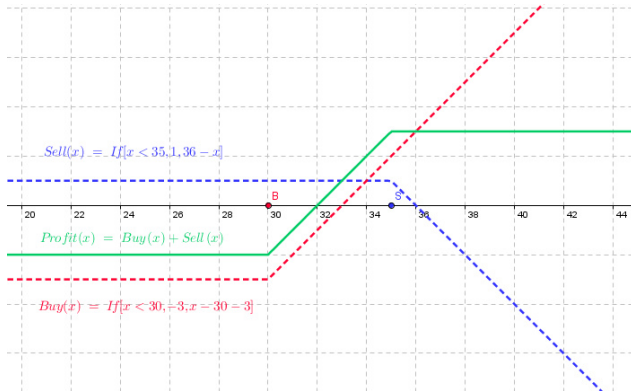


Fig. 1 Graph of overall profit function for bull call spread

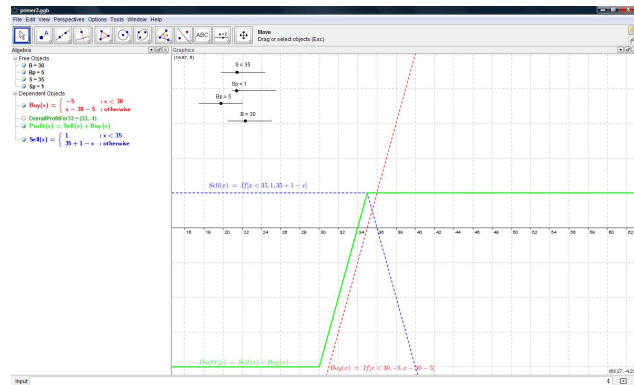


Fig. 3 The graph of overall profit function for increased B_p

Here is a chart of traders profit/loss depending on the actual price of underlying asset (see Table 1). The corresponding graph is given in Fig. 1.

From the graphs (see Fig. 2 and Fig. 3) we can conclude that the higher the B_p the lower the overall profit.

Example 4.2. A trader constructs a bear spread by selling a \$4 call with \$30 strike and buying a \$2 call with \$35 strike.

- When the overall profit will be even break?
- Considering the strike prices being as given, for what values of selling and buying premium the trader will make no loss?

Here is a chart of his profit/loss depending on the actual price of underlying asset (see Table 2).

On the other hand, overall profit is sum of profit made from selling and buying an option, therefore we used GeoGebra to find a function of overall profit and we came to the same result as in the chart (see Fig. 4).

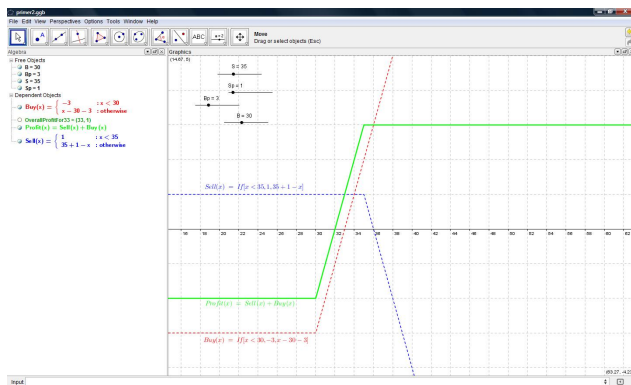


Fig. 2 Initial graph of overall profit function

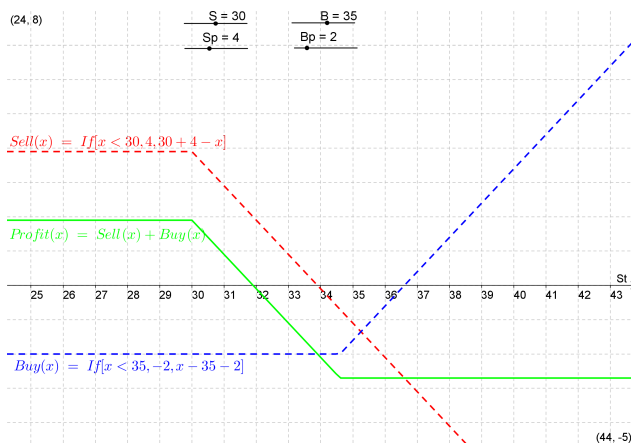


Fig. 4 The graph of overall profit function for bear call spread

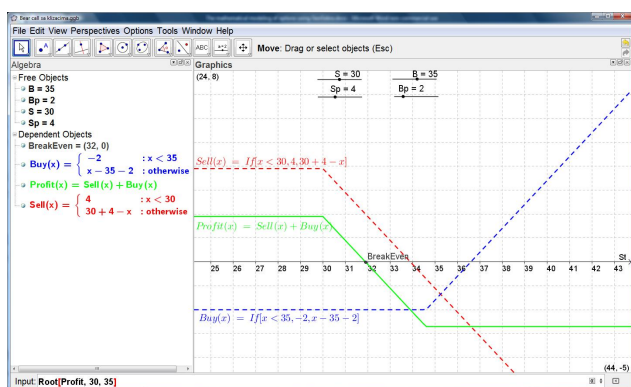


Fig. 5 The graph of overall profit function with break even point

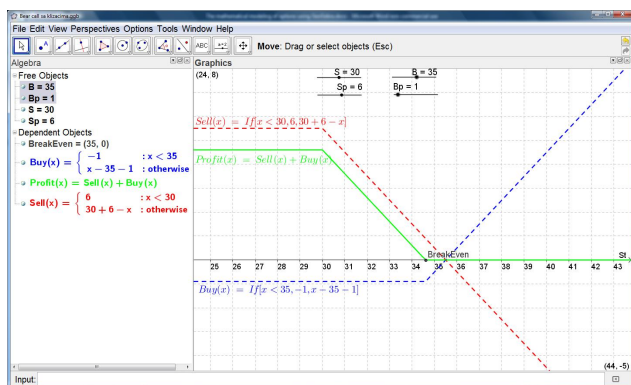


Fig. 6 The graph of overall profit function without any loss

- Using GeoGebra we created the overall profit function with four sliders: B, S, Bp, Sp. We adjusted sliders to the values given in the Example 2. Using $\text{Root}(\text{Profit}(x), 30, 35)$, we conclude that break even point is 32 – if actual price of underlying asset reaches \$32, traders profit will be \$0 (see Fig. 5).
- By adjusting Bp and Sp we came to the conclusion that trader will make no loss if premium for sold option is \$6 and for bought \$1 (Fig. 6). By analyzing some more examples we came to conclusion that whenever $B - S = S_p - B_p$ the profit will be positive.

5. DISCUSSION/CONCLUSIONS

We presented application of GeoGebra in the field of trading, specifically option trading, for which this product showed great application ability. The feature that we used the most in this contribution is slider, which allows us to animate the graph and examine its motion under specific circumstances. Furthermore, sliders properties enable us to choose increment as well as minimum and maximum of slider, which differ depending on the level of accuracy. These aspects of GeoGebra helped us come to conclusion that profit and loss in both bear and bull spread strategies are limited no matter the parameters, although combination of parameters affects the graph.

From examples analyzed we can conclude that with smart choice of parameters, in other words using smart investing, profit can be strictly positive.

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BIOGRAPHIES

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Nataša Džaleta was born on 25. 3. 1990. She is a student of Applied Mathematics at the Department of Mathematics and Informatics of the Faculty of Science at the University of Novi Sad. She has enrolled master studies this year. She speaks english (CPE) and spanish (B2).