JOINT EVALUATION OF NONLINEAR DISTORTION EFFECTS AND SIGNAL METRICS IN OFDM BASED TRANSMISSION SYSTEMS

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ABSTRACT

In this contribution, we provide a theoretical characterisation of nonlinear distortion effects in orthogonal frequency division multiplex (OFDM) either in the downlink as well as in the uplink scenario. Previous papers presented on this topic considered OFDM signal as a Gaussian distributed mostly. It will be shown that the discussion must be strictly distinguished in case we observe the downlink or the uplink OFDM scenario. In the uplink scenario, where usually low number of subcarriers are employed for the transmission and Gaussianity does not hold, poorer performance results with than that of downlink scenario are achieved. This effect will be properly explained by theoretical framework and proved by the computer simulations. Moreover, the basic properties of commonly used signal metrics for the predictions of nonlinear distortion in OFDM are introduced.

Keywords: *Cubic metric, nonlinear distortion, OFDM, Peak to average power ratio*

1. INTRODUCTION

When talking about modern digital transmission techniques for the next generation of mobile communication systems, OFDM is potential commonly agreed candidate. Despite of its great benefits, OFDM is also characterised by great sensitivity to nonlinear distortion effects caused by high power amplifiers (HPA). Due to this fact, it is inevitable to look for the effective solutions that are capable of mitigating this phenomena.

The nonlinear distortion (NLD) caused by HPA at the transmitter side creates some interference both inside and outside of the signal bandwidth. The in-band component determines a degradation of the system bit error rate (BER), whereas the out-of-band component affects adjacent frequency bands. In many applications, out-of-band radiation might become intolerable even when BER degradation is still acceptable.

There has been many research concerning NLD and its effects in OFDM signal. Some of the early work on this topic was done in [1] for baseband discrete multitone modulation (DMT), and in [2] for passband OFDM. All this papers assume only large number of subcarriers for transmission and operating point of HPA close to saturation. In such a case, OFDM signal as well as NLD are Gaussian distributed and as a result, OFDM signal at the output of HPA can be described by the scaled replica of the input signal plus an uncorrelated distortion term. The complex scaling term which is responsible for uniform rotation and attenuation of signal constellation can be easily compensated at the receiver side through the channel equalisation block. Distortion term has AWGN character and introduces the clouding of the signal constellation [3]. The result of this fact is that BER increase rapidly. However, the discussion above is valid only if the condition of employing large number of subcarries is fulfilled and HPA operates close to it's saturation. If the operating point is far from saturation, NLD behaves as a rare-event impulsive noise and can not be modelled as Gaussian anymore [4]. Therefore, it is strictly recommended to distinguish when the downlink of OFDM is investigated, where typically large number of subcarriers are adopted and uplink, where only few subcarriers are used for transmission.

Furthermore, we provide in this paper comparative study of mostly used signal metrics that are related to the nonlinear effects such as peak to average power ratio (PAPR) and cubic metric (CM). Our aim is to show their ability to predict the effects of the nonlinearity in the OFDM transmission systems. Based on the research carried out recently, it is still not entirely known how good the PAPR measure is related to effects of the nonlinearity and following this fact, we sketch the basic example of doing so as well.

Based on the above mentioned discussion, we can claim that nonlinear amplification in OFDM transmission systems has a crucial influence on overall performance and therefore its effects must be taken into account very carefully. Regarding this, the three folds are the aim of this contribution. In the first part of this paper, we provide the theoretical framework concerning NLD. The stress is focused on analysis of nonlinear distortions introduced by HPA on Gaussian OFDM signal. It is worth bearing in mind, that this discussion is not valid for non-Gaussian signals, which is explained as well. In the second part of this paper, BER performance for non-Gaussian OFDM signals is introduced and described by means of computer simulations. Last part of this paper is devoted to the comparative study of the widely accepted signal metrics for the assessment of nonlinear distortion in OFDM based transmission systems. The simulation results presented in this section will suggest, that CM predicts the impact of nonlinear distortion on overall BER performance in more precise way, than it is in the PAPR case.

2. OFDM OVERVIEW

OFDM transmit signal is the sum of N independent subsymbols (tones) with equal bandwidth and frequency separation $1/T$, where T is time duration of OFDM symbol. The $m-th$ group of encoded bits is mapped into the complex valued OFDM vector of QAM constellation points, $\mathbf{X}^m = [X_0^m, \ldots, X_{N-1}^m]$ and the continuous time representation of the single multicarrier symbol is given by [5]:

$$\mathbf{x}_{CP}^m(t) = \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{k=N/2-1} X_k^m e^{j2\pi kt/T} \tag{1}$$

where m is a symbol index and X_k^m is is the QAM value of k-th subsymbol or tone. The periodic extension of the symbol over the interval $[-T_{CP}, 0]$ is the cyclic prefix (CP) which simplifies the equaliser design in the presence of multipath fading.

In practise, OFDM signals are generated using Inverse Discrete Fourier Transform (IDFT). The resulting T/N-spaced discrete time vector $\mathbf{x}^m = [x_0^m, \ldots, x_{N-1}^m] = IDFT(\mathbf{X}^m)$ is given by:

$$\mathbf{x}^m = \frac{1}{\sqrt{N}} \sum_{k=0}^{k=N-1} X_k^m e^{j2\pi kn/N} \tag{2}$$

In this paper, the discrete time indexing $[n]$ denotes Nyquist rate samples. In order to avoid aliasing and the out-of-band radiation into to data bearing tones, the oversampling of the original signal $x[n]$ may be needed [6]. We will introduce the notation $x[n/L]$ to denote oversampling by factor L. In the simulation presented in this paper, oversampling factor of 4 has been applied.

3. CHARACTERISATION OF DECISION VARIABLES AT THE DEMODULATOR STAGE

In order to be able to describe statistical behaviour of nonlinear distortion, we first describe the signal at the input of HPA, and in the next we exploit the signal at the output of HPA. Finally, we investigate the signal in the presence of AWGN at the input of the baseband modulator.

3.1. Signal at the input of HPA

In general, OFDM signal is a sum of many independent subcarriers, that are modulated by baseband QAM symbols. Without loss of generality, we can assume, that baseband QAM symbols are identically distributed random variables. According to the central limit theorem, it is reasonable to assume that OFDM signal is Gaussian distributed if large number if subcarriers is employed for transmission. As a result, OFDM signal envelope follows Rayleigh distribution as [7]:

$$f_X(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} \tag{3}$$

Since the envelope of OFDM signal follows Rayleigh distribution, it is apparent, that the envelope significantly fluctuates in time. It is worth to realise, that the signals with different amplitudes will be affected by the nonlinearity differently, following the probabilistic distribution of the OFDM signal. This fact has crucial influence on the overall performance of OFDM transmission system [8].

3.2. Signal at the output of HPA

Now, we focus our attention on the nonlinear block of HPA. Following mathematical derivation are derived in this section by the application of Bussgang theorem (*Appendix A*) [9]. The output signal $y(t)$ of memoryless HPA can be described as follows:

$$y(t) = \alpha x(t) + d(t) \tag{4}$$

where $\alpha = R_{xy}(\tau_1)/R_{xx}(\tau_2)$ is complex gain, R_{xy} denotes crosscorrelation function of input and output signal and R_{xx} denotes autocorrelation function of the input signal. Note that α is independent on particular OFDM symbol realisation $[X_0, \ldots, X_{N-1}]$ and remains constant during entire transmitting process [4]. The scaling factor α is responsible for the attenuation and rotation of the constellation, which can be easily compensated at the receiver by introducing correction factor $\alpha/|\alpha|^2$. However, the distortion term $d(t)$, which is responsible for both the clouding and the out-of-band radiation can not be compensated by conventional receivers [4]. It is easy to show that $d(t)$ is uncorrelated with the input signal $x(t)$:

$$\begin{aligned} R_{xd} &= E[x(t) * (y(t+\tau) - \alpha x(t+\tau)) \\ &= R_{xy}(\tau) - \alpha R_{xx}(\tau) = 0 \end{aligned} \tag{5}$$

3.3. Output vector of the Fast Fourier Transform (FFT) block

This part of the paper determines the BER performance in nonlinear AWGN environment. The signal at the output of FFT is given by [4]:

$$R_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{LN-1} r_n e^{-j2\pi kn/LN}, k = 0, \ldots, N-1 \tag{6}$$

where r_n is the input signal of FFT. In general R_k consists of the useful part of the signal, noise components and nonlinear distortion. This fact can be expressed as [4], [3]:

$$R_k = \alpha S_k + D_k + W_k \tag{7}$$

where D_k is nonlinear distortion term in expressed in frequency domain and W_k is AWGN component.

Let us analyse nonlinear distortion term in our further discussion. Nonlinear distortion at the output of FFT can be expressed as [3], [4]:

$$D_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{LN-1} d_n e^{-j2\pi kn/LN} \tag{8}$$

Note that nonlinear distortion term D_k is composed as the sum of N identically distributed random variables, so we can assume it to be complex Gaussian distributed. In [3] it was shown that even the certain parts of the sum are correlated, after all, D_k might be considered as a complex Gaussian.

As it is clear from above mentioned discussion, both W_k and D_k follows Gaussian distribution which lead us to the formulation of the signal to noise-plus-distortion ratio (SNDR) as follows [4]:

$$SNDR = \frac{|\alpha|^2 E[|S^2|]}{\sigma_W^2 + \sigma_D^2} \tag{9}$$

where σ_D^2 and σ_W^2 is the variance of nonlinear distortion term and variance of Gaussian noise, respectively.

4. INPUT BACK-OFF AND OPERATING POINT OF HPA

The operating point of nonlinearity is defined by the so called input back-off (IBO) which corresponds to the ratio between the saturated and average input powers [10]:

$$IBO = \frac{P_{in}}{P_{sat}} \tag{10}$$

where $P_{sat} = A_{sat}^2$ represents the saturation power and $P_{in} = E|x(t)|^2$ represents the mean power of the input signal $x(t)$ of HPA. Small values of the *IBO* causes the amplifier operation point to be near the saturation. In this case a good HPA efficiency is achieved, but as a consequence the HPA output signal will be highly distorted. Keep in mind that if high *IBO* is applied at the transmitter side, OFDM transmission is strongly inefficient, HPA operates far from saturation point and nonlinear distortion behaves as a rare-event impulsive noise and follows no more Gaussian distribution [4].

In order to illustrate the above mentioned fact, the appropriate computer simulations have been arranged. Within these simulations, OFDM signals with $N = 256$ subcarriers, operating at Rapp model of HPA [13], is presented. Fig. 1 represent BER of OFDM as a function of E_b/N_0. On one hand the simulation results match the analytical results presented in [4] for low values of *IBO* almost perfectly ($IBO = 2dB, 4dB, 6dB$), on the other hand, with increasing value of ($IBO = 10dB, 12dB$), the differences between analytical results and results obtained by computer simulation are increasing. In these cases, the HPA is more linear, less distortion is introduced and therefore distortion term introduced by HPA becomes less Gaussian distributed. This observation clarifies the strong differences between theoretical results derived by assuming the Gaussian distribution of distortion term and experimental results for high IBO parameters adopted for transmission.

In Fig. 2, power density spectrum of OFDM signal for different *IBO* parameters is shown. The simulation results confirm the theoretical expectations, the higher *IBO*, the lower out-of-band radiation is presented. However during the design process, one has to find trade-off between BER, power efficiency and out-of-band radiation [11].

5. PERFORMANCE OF NON-GAUSSIAN OFDM SIGNAL

The mathematical formulas presented above have been derived under condition of Gaussianity of OFDM signal. However, when low number of subcarriers is adopted, Gaussianity does not hold. It was shown by means of normalised curtosis in [4] , that OFDM signal with low number

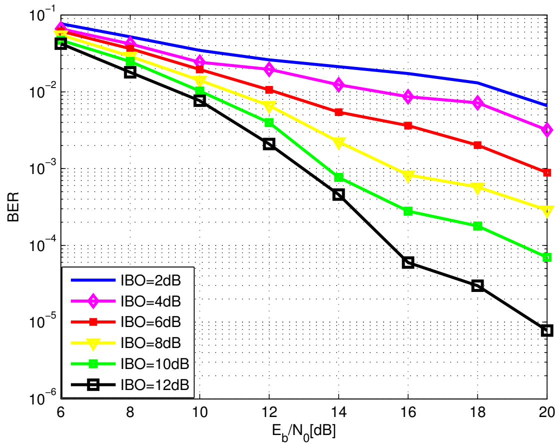

Fig. 1 BER vs. E_b/N_0, different values of *IBO*

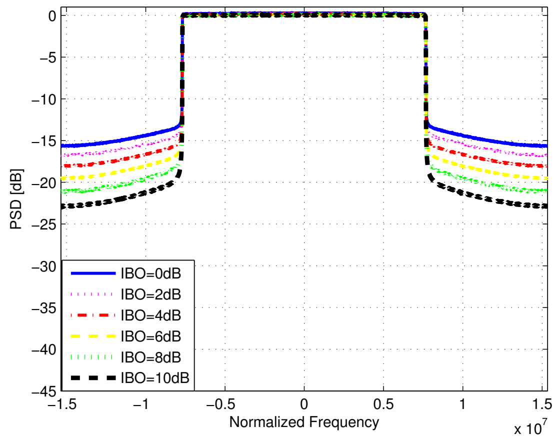

Fig. 2 Power density spectrum, different values of *IBO*

of subcarriers follows sub-Gaussian distribution. The signal at the output of HPA is given in this case as [4]:

$$y^i(t) = \alpha^i s^i(t) + d^i(t) \tag{11}$$

where α^i is a complex gain factor that depends on particular OFDM symbol realisation $[X_0^i, \ldots, X_{N-1}^i]$ and $d^i(t)$ is the distortion term. In such case, the complex gain α^i is different for different OFDM symbols. As it was described in [4], sub-Gaussian signals exhibit much worse BER performance results in AWGN channel, than signals that follows Gaussian distribution. This fact is illustrated by Fig. 3, where BER vs E_b/N_0 for different number of subcarriers is illustrated. In this simulation, OFDM system using 16-QAM and Rapp model operating at $IBO = 8dB$ is considered. As it is clear from this figure, the lower number of subcarriers is employed for transmission, the higher BER is achieved. This fact is crucial mainly in the uplink of OFDM, where only few subcarriers are usually adopted. In order to mitigate this problem, 3^{rd} Generation Partnership Project (3GPP) introduced single carrier frequency division multiplex (SC-FDMA) in an upcoming 4G wireless communication uplink system instead of OFDM [12]. SC-FDMA is characterised by considerable lower envelope

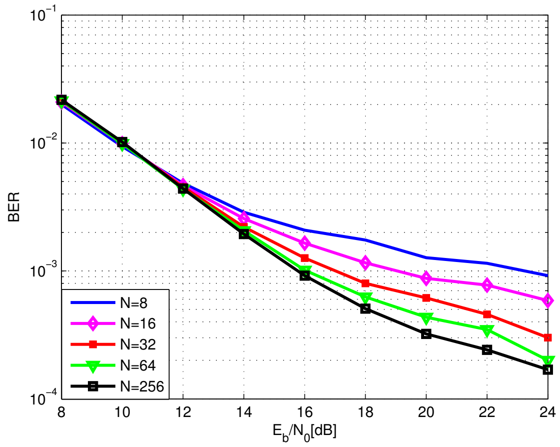

Fig. 3 BER vs. E_b/N_0, different number of subcarriers

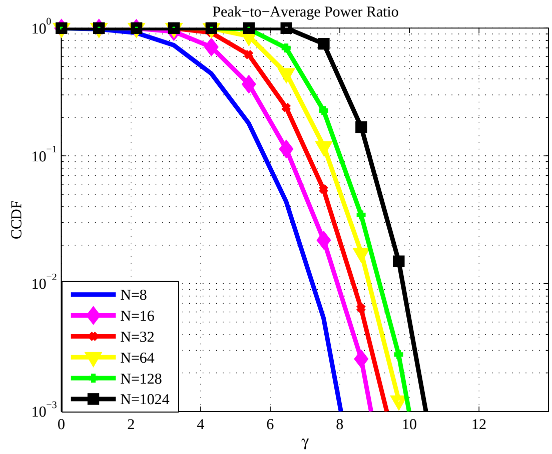

Fig. 4 CCDF of PAPR - different number of subcarriers

fluctuation which results in higher robustness against NLD and what is very important, NLD remains constant with changing number of subcarriers [4], [12].

6. OFDM SIGNAL METRICS

In order to predict the nonlinear effects on the OFDM transmitted signal, it is necessary to introduce a common signal metric platform. PAPR has been widely accepted as an objective signal metric in the process of investigation of nonlinear effects in OFDM. Let $\mathbf{x}^m = [x_0^m, \ldots, x_{N-1}^m]$ be the m-th OFDM symbol generated in (2). PAPR of $\mathbf{x}^m$ is defined as the ratio between its peak and average power as follows:

$$PAPR(\mathbf{x}^m) = \frac{||\mathbf{x}^m||_\infty^2}{E(||\mathbf{x}^m||^2)/N} \tag{12}$$

where $||.||_\infty$ denotes euclidean norm and E is expectation operator. To evaluate the PAPR characteristics of the OFDM signal, it is necessary to compute the probability that the PAPR of an OFDM symbols exceeds a given threshold. This can be accomplished by evaluating complementary cumulative density function (CCDF) over all OFDM symbols. CCDF characterise the probability that randomly generated OFDM symbol exceeds the given threshold γ. In Fig. 4 we can see CCDF of OFDM system with different number of subcarriers. The simulation curves suggest the fact, that with increasing number of subcarriers, OFDM envelope fluctuation increase inherently.

We can observe mayor drawback from *PAPR* definition introduced in [12]. As it is clear, PAPR takes into account only highest peaks over OFDM modulated symbol, the other high peaks are not taken into account directly and are only included in [12] in the form of average power of transmitted OFDM symbol. It was discussed in [6], that also other high (minor) peaks within OFDM symbol are relevant in the process of prediction of nonlinear distortion and hence nonlinear distortion evaluation based on PAPR signal metric is feasible to be improved.

In order to alleviate the problems mentioned above, 3GPP has introduced Cubic Metric (CM) to quantify the envelope fluctuation of OFDM modulated signals.

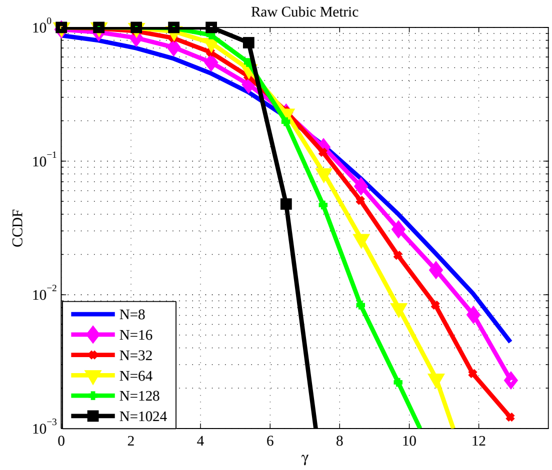

Fig. 5 CCDF of RCM - different number of subcarriers

CM is defined as follows:

$$CM|_{dB} = (RCM|_{dB} - RCM_{ref}|_{dB})/K \tag{13}$$

where RCM is called the raw CM, defined for OFDM signal $x(t)$ as follows:

$$RCM(x(t))|_{dB} = 20log\left[rms\left[\left(\frac{|x(t)|}{rms\left[x(t)\right]}\right)^3\right]\right]. \tag{14}$$

$RCM_{ref}|_{dB}$ is the RCM of the reference signal and K is a constant defined by 3GPP in order to meet specific requirements of the OFDM transmission [14]. In this paper, *RCM* of each discrete OFDM symbol is also independently consider as:

$$RCM(\mathbf{x}^m) = \sqrt{E\left[\left(\frac{|\mathbf{x}^m|^2}{P_s}\right)^3\right]}, \tag{15}$$

so one can obtain the *RCM* characteristics of the OFDM signal by means of CCDF. In (15), $P_s = \sigma^2$ is the average power of the OFDM symbol, which is assumed to be constant for all OFDM symbols. CCDF characterise in this case again the probability that randomly generated OFDM symbol exceeds the given *RCM* threshold γ. Typical example of RCM CCDF for different number of subcarriers is sketched in Fig. 5.

Results presented in Fig.3-Fig.5 introduce some contradiction between general expectations (lower PAPR, better BER performance) and simulation results. As we can observe, with increasing number of subcarriers PAPR increase, while BER performance degrades. The reason for this observation and mathematical analysis supporting this fact has been introduced in [4] very recently. This fact has not been taken into account sufficiently enough and therefore it is still widely believed that high PAPR means higher influence of nonlinear distortion. By contrast, CM decrease with increasing number of subcarriers. As it can be seen, there is a noticeable distinction in prediction of nonlinear distortion between two mostly used signal metrics, PAPR and CM. From the BER point of view, we can claim that CM is more suitable for the prediction of the influence of nonlinear distortion, since it decreases with increasing number of subcarriers as well as BER does.

7. CONCLUSIONS

Nonlinear amplification might be a very difficult challenge, unless the theoretical aspects of nonlinear distortion are not explained properly. This fact motivates the authors to provide the theoretical discussion of the nonlinear effects in OFDM. In the first part of the paper theoretical approach to the evaluation of nonlinear distortion effects of OFDM signal in AWGN channel is presented. To build this procedure, Bussgang theorem has been adopted in mathematical formulation. However, it was shown, that for low number of subcarriers, Bussgang theorem is not valid and OFDM signal becomes following sub-Gaussian distribution. In a second part of this paper, we observed, that BER degradation due to NLD is related to the number of subcarrier. The lower number of subcarriers is, the higher BER degradation occur. From this point of view, OFDM uplink transmission is much more vulnerable to nonlinear amplification than the downlink, where Bussgang theorem holds. Therefore we highly recommend to use in this case particular available methods that mitigate this harmful nonlinear effects either on transmitter, or receiver side. Last part of this paper deals with well-known signal metrics for evaluating the sensitivity of OFDM signal to nonlinear distortion. By means of CCDF it was shown, that CM is more suitable for the prediction of nonlinear impact from BER point of view than PAPR. These results are in accordance with 3GPP specifications introduced recently.

APPENDIX

Bussgang theorem

For two Gaussian signals $x_1(t)$ and $x_2(t)$, the cross-correlation function taken after one of them (e.g. $x_2(t)$) has undergone nonlinear amplitude distortion ($R_{x_1y_2}$) is identical except for a factor of proportionality α, to the cross-correlation function taken before the distortion ($R_{x_1x_2}$):

$$R_{x_1y_2}(\tau) = \alpha R_{x_1x_2}(\tau) \quad (16)$$

Notice that if $x_1(t) = x_2(t)$ then it follows that the cross-correlation between input and output signals of the nonlinearity is identical, except for a factor of proportionality α, to the autocorrelation of the input signal, that is

$$R_{xy}(\tau) = \alpha R_{xx}(\tau) \quad (17)$$

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