# PASSIVITY BASED CONTROL WITH ORIENTATION OF THE FLUX OF A PERMANENT MAGNET SYNCHRONOUS MOTOR WITHOUT MECHANICAL SENSOR

# Baghdad BELABBES\*, Mohamed K. FELLAH\*, Abdelkader.LOUSDAD\*\*, Abdelkader MEROUFEL\*, Mohamed ABID\*

\*Intelligent Control and Electrical Power Systems (I.C.E.P.S), University of Djillali Liabes BP 98 Sidi belabbes, Algeria \*\*Faculty of Engineering Sciences, Mechanical Engineering Department, Laboratory of Mechanics of Materials and Reactive Systems, University of Sidi Bel Abbes, BP 89, Cité Ben M'Hidi, Sidi Bel Abbes, 22000–Algeria, Tel /Fax: (213) 048 54 4100 E-mail: Belabbesbag@yahoo.fr, mkfellah@yahoo.fr, a\_lousdad @yahoo.com, ameroufel@yahoo.fr

#### ABSTRACT

The aim of the present paper is the study of the behavior of passivity based control and difficulties due to synthesis for various operating conditions of a synchronous motor with a permanent magnets. The study takes into account the guarantee of satisfactory static and dynamic performance. It also allows the system to be insensitive to disturbances and uncertainties on the parameters. A number of estimation techniques have been developed to achieve speed and position permanent magnet synchronous motor (PMSM) drives without mechanical sensor. Most of them suffer from variation of motor parameters such as the stator resistance, stator inductance and torque constant. Also, it is known that conventional linear estimators are not adaptive variations of the operating point in a nonlinear system. The sliding mode technique has shown promising results when estimating or controlling nonlinear systems. This paper presents a sliding mode observer (SMO) estimating the position and speed of a permanent magnet synchronous motor (PMSM) to achieve sensorless drive system. The effect of variations of motor parameters such as torque constant, stator resistance and stator inductance on the position and/ speed estimations, over a wide speed range, have been studied. Compared to other methods, the observer is more robust to operating conditions and parameter uncertainties. Simulations show that the observer is robust.

**Keywords**: Passivity Based Control (P.B.C), F.O.C (Field Oriented Control), PMSM (Permanent magnet synchronous motor), sensorless control, SMO (sliding mode observer).

# 1. INTRODUCTION

The use of electrical machines is expanding rapidly due to the offered performance.

The control of machines is the primary concern of control theory research. In fact, an electrical machine is characterised by a non linear behavior. Additing to that the major difficult tasks to be executed which require a higher precision under rapid trajectories. In order to meet performance criteria always in increase, algorithms of control more and more complex are developed. The progress of computers allow to implement these new strategies in industry. In classical control theory, the linear models are considered. The non-linear equations are linearized with a linear system if they can be linearized in order to determine the control laws. The derived control laws from this approach are adequate and sufficient in many practical application, but in certain cases the linear approaches are not sufficient. Thus, a theory for non linear system is necessary. However, the non linear theory for general systems is complicated and seldom worthy in technological applications. But, from the accomplished works these last three decades, aiming to improve performance, advanced research had allowed emergence of new non linear control techniques for electrical machine application. In this context a control technique requiring the perfect knowledge of the model will be presented. The control in consideration is the passivity control developed by Sadegh and Horwitz [1] issued from Slotine and Li's work [4], it uses essentially.

For controlling a permanent magnet synchronous motor (PMSM), the Lagrangian structure of mechanical systems in order to make a decreasing Lyapunov's function. It is necessary to know the position of the rotor. The stator currents of the PMSM are controlled to generate constant torque using the rotor position signal. Encoders or revolvers have been used for sensing the position. These position sensors, however, make the motor expensive and mechanically unreliable. Position and speed estimation techniques to eliminate the encoders and revolvers have been studied.

This paper describes the new sensorless control of the PMSM using a sliding mode observer. The proposed methodology incorporates the Lyapunov's algorithm to estimate the rotor speed and the stator resistance so that it can overcome the problem of sensitivity in the face of motor parameter variation. Also, without any mechanical information, the rotor speed is obtained from adaptive scheme. Results of simulation are carried out to verify the feasibility of the sensorless control for the PMSM [5].

# 2. GENERAL FORMULATION OF THE PASSIVITY CONTROL

It is possible to distinguish two fundamental steps while using passive control for a given system. The system modelling is put under the EL formalism and its (possible) passivity is used to create relations describing the stabilizing control. From these relations and by using a variety of techniques (control with variable structure a control based on average representation); the dynamic of the corrector is computed (if it exists) and the control value. In order to achieve the stabilized control, the EL properties are used which exist in all the machine circuits. The first property states that any control circuit can be represented under EL formalism [3].

### 2.1. Definition of passivity

The basic idea of the passivity consists in shaping the total energy of the system then in adding a damping term. EL equation allows to obtain easily the formulation after having formulated the total energy of the system; it is modified to desired (minimum) value. The system converge towards this minumum also if the control able to inject an additif dissipative term to the system, then the convergence to the derived state can be improved with respect to that obtained by natural dissipation given by the system .

In this subsection the definitions of passivity are those given in [1]. A continuous systems with input u and an output y is referred to be dissipative if a function V(t) > 0 exists and verifies the following equation.

$$\dot{V} = y^{T}(t)u(t) - h(t)$$
; with  $\int_{0}^{t_{f}} h(t)dt > 0$  (1)

where  $y^{T}$  represents the transpose vector of y if  $h(t) \ge 0$ the system is said to be dissipative.

In reality one may notice that  $y^{T}(t)u(t)$  represents a power and its integration represents energy. The system is passive if the input energy is more important than that it gives to the environment (of the output) one may know that a passive system is necessarily stable [3].

### **2.2.** Passivity and dissipativity of a system [2]:

### **Function of storage**

**Theorem 1** Let us suppose that there is a continuous function  $V(t) \ge 0$  such as:

$$V(T) - V(0) \le \int_{0}^{T} y^{T}(t)u(t)dt$$
(2)

for all functions u and for all  $T \ge 0$ . Then the system of input u(t) and output y(t) is passive. Moreover let us suppose, that there are two constants  $\delta \ge 0$  and  $\epsilon \ge 0$  such as:

$$V(T) - V(0) \le \int_{0}^{T} y^{T}(t)u(t)dt - \delta \int_{0}^{T} u^{T}(t)u(t)dt - \varepsilon \int_{0}^{T} y^{T}(t)y(t)dt \quad (3)$$

for all functions u and all V(0) for  $T \ge 0$  then the system is strictly passive in input if  $\delta > 0$ , strictly liability at output if  $\epsilon > 0$ , and strictly liability if  $\delta > 0$  and  $\epsilon > 0$ .

One calls "function of storage" of the input system u(t) and output y(t), the function V (t) of theorem 1.

Theorem 2 (Theorem of the passivity)

Let us suppose that two  $\Sigma_e$  systems and  $\Sigma_m$  in closed loop (respectively of input  $u_1$  and  $u_2$ , output  $y_1$  and  $y_2$ )(Fig. 1) are pseudo strictly passive, is stable with finished gains if:

$$\int_{0}^{T} y_{1}^{T}(t)u_{1}(t)dt + \beta_{1} \ge \delta_{1} \int_{0}^{T} u_{1}^{T}(t)u_{1}(t)dt + \epsilon_{1} \int_{0}^{T} y_{1}^{T}(t)y_{1}(t)dt$$

$$\int_{0}^{T} y_{2}^{T}(t)u_{2}(t)dt + \beta_{2} \ge \delta_{2} \int_{0}^{T} u_{2}^{T}(t)u_{2}(t)dt + \epsilon_{2} \int_{0}^{T} y_{2}^{T}(t)y_{2}(t)dt$$
(4)

with : 
$$\delta_1 \ge 0, \epsilon_2 \ge 0$$
;  $\delta_1 + \epsilon_1 \ge 0$  et  $\delta_2 + \epsilon_2 \ge 0$ 



Fig. 1 Interconnection of two under passive systems.

### 2.3. Relation between stability and dissipativity¶

### 2.3.1. Linear system

In order to illustrate the relationship between the stability and dissipativity and for the sake of simplicity, an open loop stable system is proposed defined as [4]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{v} = \mathbf{C}\mathbf{x}$$
(5)

for any stable system of the form

$$= Ax (6)$$

we have

ż

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{O} \tag{7}$$

the quadratic function

$$\mathbf{V} = \frac{1}{2} \mathbf{x}^{\mathrm{T}} \mathbf{P} \mathbf{x} \tag{8}$$

is the Lyapunov's function for the system defined by (6) because

$$\dot{\mathbf{V}} = -\frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} \le \mathbf{0} \tag{9}$$

the Lyapunov's function (7) is verified for the system described in (5) but it is only linked to the system stability characterized by the matrix A and is independent of the input u. Using the derivative of Lyapunov's function (8) with substitution of (5) we obtain:

$$\dot{\mathbf{V}} = -\frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{B}\mathbf{u}$$
(10)

The equation (10) is of the form given in (1) this leads to a dissipative transfer between  $x^{T}PB$  and u, more, if B and C a related such that.

$$\mathbf{C} = \mathbf{B}^{\mathrm{T}} \mathbf{P} \tag{11}$$

the equation (10) becomes:

$$\dot{\mathbf{V}} = -\frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{y}^{\mathrm{T}}\mathbf{u}$$
(12)

### 2.3.2. Multivariable non-linear system

Let's consider a multivariable non linear system having the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{t}) + \mathbf{g}(\mathbf{x}, \mathbf{t})\mathbf{u}$$
  
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{t})$$
(13)

with  $x \in \Re^n$ ,  $y \in \Re^m$ ,  $u \in \Re^m$ , f, g, h are continuous with and slowly varying with x. assuming that f(0,t) = 0, and h(0,t) for any  $t \ge 0$ 

the system described by (10) is said to be passive if there exists a scalar function which is continuous and positive V:  $\Re^n \times \Re_+ \rightarrow \Re_+$ 

which satisfies V(0,t) = 0,  $\forall t \ge 0$ , such that:

$$\int_{t_0}^{t} y^{\mathrm{T}}(\sigma) u(\sigma) d\sigma \ge V(x(t), t) - V(x(t_0), t_0)$$
with  $t \ge t_0 \ge 0$ 
(14)

### 3. PASSIVITY BASED MATHEMATICAL DESCRIPTION OF THE FEEDBACK STATE

### 3.1. PMSM model current control

The PMSM model adopted in this case of control is that represented in the dq referential expressed by the following equations [3]:

$$L_{dq}\frac{dI_{dq}}{dt} + R_{dq}I_{dq} + p\omega_m \Im L_{dq}I_{dq} + p\omega_m \varphi_f = V_{dq}$$
(15)

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{\mathbf{m}} + \mathbf{B}\boldsymbol{\omega}_{\mathbf{m}} = \boldsymbol{\tau} - \boldsymbol{\tau}_{\mathbf{L}} \tag{16}$$

with 
$$\tau = -\frac{3}{2} p \left( (\mathbf{L}_d - \mathbf{L}_q) \mathbf{i}_d \mathbf{i}_q + \phi_f \mathbf{i}_q \right)$$
 (17)

The equation (15) and (17) can be expressed with respect to the flux under the form:

$$\dot{\psi}_{dq} + p\omega_m \Im \psi_{dq} = V_{dq} - R_{dq} I_{dq}$$
(18)

$$\tau = -\frac{3}{2} p \psi_{dq}^{T} \Im \mathbf{I}_{dq}$$
(19)

with 
$$\psi_{dq} = L_{dq}I_{dq} + \phi_f$$
 and  $\Im = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  (2)

 $\mathbf{R}_{\mathbf{dq}} = \mathbf{R}_{\mathbf{a}}$ : stator resistance,  $\mathbf{L}_{\mathbf{dq}} = \mathbf{L}_{\mathbf{d}} = \mathbf{L}_{\mathbf{q}} = \mathbf{L}$ : stator inductance. The mathematical subsystem is considered as a passive disturbance from CBP point of view. It is assumed that the stator current  $I_{dq}^{\bullet}$  are the control input, the M.S.A.P is controlled in currents.

$$\mathbf{I}_{dq}^{\bullet} = (\mathbf{i}_{d}^{\bullet} \ \mathbf{i}_{q}^{\bullet})^{\mathrm{T}}$$
(21)

thus two statoric current control loops given by the relations.

$$V_{d} = k_{dp}(i_{d}^{\bullet} - i_{d}) + k_{di} \int_{0}^{t} (i_{d}^{\bullet} - i_{d}) d\sigma$$
 (22)

$$V_{q} = k_{qp}(i_{q}^{\bullet} - i_{q}) + k_{qi} \int_{0}^{t} (i_{q}^{\bullet} - i_{q}) d\sigma$$
(23)

with 
$$\{k_{dp}, k_{di}, k_{qp}, k_{qi}\} \subset \mathfrak{R}^{\bullet+}$$
 (24)

The two PI control loops an interior and facing current  $I_{dq}$  of the motor to follow the reference  $I_{dq}^{\bullet}$ .

The determination of their gain is made by classical methods (pole placement) or advanced method (PI flou). In this work the method of placement of poles is used. Assuming that the two PI loops correctly achieve the task due to a convenient choice of their gains by using equations (18) (19) (22) et (23), the dynamics of PMSM current control is reduced to the following model

$$\dot{\psi}_{dq} + p\omega_m \Im \psi_{dq} = -R_{dq} I_{dq}$$
(25)

$$\mathbf{J}\dot{\boldsymbol{\omega}}_{m} + \mathbf{B}\boldsymbol{\omega}_{m} = -\frac{3}{2}\mathbf{p}\boldsymbol{\psi}_{dq}^{T}\Im\mathbf{I}_{dq}^{\bullet} - \boldsymbol{\tau}_{L}$$
(26)

the developed torque :

$$\tau = -\frac{3}{2} p \psi_{dq}^{T} \Im I_{dq}^{\bullet}$$
(27)

) The PBC is developed form the model (25) (26) (27) of the current control PMSM.

### 3.2. Passivity properties of PMSM in dq model

The property of passivity in dq model of PMSM demonstrated in a different manner than that established we can formulate the following lemma:

8) **Lemma**: the input – output relation  $I_{dq}^{\bullet} \mapsto \psi_{dq}$ 

describing the electrical part (25) of input  $I_{dq}^{\bullet}$  and output  $\psi_{dq}$  is passive.

**Proof**: Passivity in open loop of the model flux in dq referential by multiplying the equation (25) on the left by  $\mathbf{R_{dq}^{-1}} \boldsymbol{\psi_{dq}^{T}}$  and knowing that  $\boldsymbol{\Im}$  is a unit matrix and antisymetric we obtain:

$$\mathbf{I}_{dq}^{\bullet T} \boldsymbol{\psi}_{dq} = -\frac{1}{2} \mathbf{R}_{dq}^{-1} \frac{\mathbf{d}}{\mathbf{dt}} (\boldsymbol{\psi}_{dq}^{T} \boldsymbol{\psi}_{dq})$$
(26)

By integrating (26) in the interval  $[0, I_{dq}]$ , and while proceeding after integration to an increase, we obtain the following inequality:

$$\int_{0}^{T_{dq}} I_{dq}^{\bullet T} \psi_{dq} d\sigma > -\frac{1}{2R_a} \int_{0}^{T_{dq}} \left\| \psi_{dq} \right\| d\sigma + \beta_f$$
(27)

with:  $\beta_f < 0$ 

From the dissipation inequality (27) and according to the passivity definition, that the electrical subsystem described by relation (25) is passive considering the mechanical part in (26) as perturbation.

# **3.3.** Design of the passivity based control current control

The PBC of the PSMS current control is developed in model (25)-(27) whose signals desired current  $I_{dq}^{\bullet}$ . The computation is made on electrical part (25) the mechanical part (26) is considered as a disturbance. During the synthesis procedure the stability in the sense of Lyapunov is developed [3].

Let the flux vector  $\psi_{dq}^{\bullet}$  the following error of this quantity

is: 
$$\mathbf{e_f} = \mathbf{\psi_{dq}} - \mathbf{\psi_{dq}^{\bullet}}$$
 (28)

The dynamic equation of the error of the flux is obtained form equations (25), (28) and give:

$$\dot{\mathbf{e}}_{\mathbf{f}} + \mathbf{p}\boldsymbol{\omega}_{\mathbf{m}}\boldsymbol{\Im}\mathbf{e}_{\mathbf{f}} = -\mathbf{R}_{\mathbf{d}\mathbf{q}}\mathbf{I}_{\mathbf{d}\mathbf{q}}^{\bullet} - (\dot{\boldsymbol{\psi}}_{\mathbf{d}\mathbf{q}}^{\bullet} + \mathbf{p}\boldsymbol{\omega}_{\mathbf{m}}\boldsymbol{\Im}\boldsymbol{\psi}_{\mathbf{d}\mathbf{q}}^{\bullet})$$
(29)

In order to give a proof for the convergence of the error of the flux  $e_f$  the Lyapunov theory of stability is used. Given the following quadratic function:

$$\dot{\mathbf{V}}_{\mathbf{f}}(\mathbf{e}_{\mathbf{f}}) = \frac{1}{2} \mathbf{e}_{\mathbf{f}}^{\mathrm{T}} \mathbf{e}_{\mathbf{f}}$$
(30)

After derivation of  $V_f$  along the trajectory (29), we obtain:

$$\dot{\mathbf{V}}_{\mathbf{f}}(\mathbf{e}_{\mathbf{f}}) = -\mathbf{e}_{\mathbf{f}}^{\mathrm{T}}(\mathbf{R}_{\mathbf{d}q}\,\mathbf{I}_{\mathbf{d}q}^{\bullet} + (\dot{\psi}_{\mathbf{d}q}^{\bullet} + \mathbf{p}\boldsymbol{\omega}_{\mathbf{m}}\,\mathfrak{T}\,\boldsymbol{\psi}_{\mathbf{d}q}^{\bullet})) \tag{31}$$

The function  $\dot{V}_f$  is negative defined if and only if the control vector  $I_{dq}^{\bullet}$  is chosen under the following form:

$$\mathbf{I}_{dq}^{\bullet} = \mathbf{R}_{dq}^{-1}(-(\dot{\psi}_{dq}^{\bullet} + \mathbf{p}\boldsymbol{\omega}_{m}\Im\psi_{dq}^{\bullet}) + \mathbf{K}_{f}\mathbf{e}_{f})$$
(32)

with 
$$\mathbf{K}_{\mathbf{f}} = \mathbf{k}_{\mathbf{f}} \mathbf{I}_2$$
 et  $\mathbf{k}_{\mathbf{f}} > 0$  (33)

# **3.3.1.** Computation of the derived flux $\psi_{da}^{\bullet}$

The PMSM working at max torque if the direct current  $i_d$  is null i.e. according to relation (20) the flux

 $\psi_d$  along the direct axis d is reduced to the  $\phi_f$  flux created by the permanent magnets. Thus, the desired flux along the d axis is chosen [3].

$$\Psi_{\rm d}^{\bullet} = \phi_{\rm f} \tag{34}$$

From equation (20) and (25) the desired torque is defined by the following relation.

$$\tau^{\bullet} = -\frac{3}{2} p \psi_{dq}^{\bullet T} \Im(\psi_{dq}^{\bullet} - \psi_{f})$$
(35)

The desired flux along the quadratic axis q; from equations (34) and (35) is defined as:

$$\Psi_{\mathbf{q}}^{\bullet} = \frac{2}{3} \frac{L_{\mathbf{q}}}{\mathbf{p}\phi_{\mathbf{f}}} \tau^{\bullet}$$
(36)

### **3.3.2.** Calculation of the desired torque $\tau^{\bullet}$

In order to compensate the disturbance of the resisting torque at the rotational speed response we propose that the machine is controlled using IP regulator.

 $\Omega_{ref} \xrightarrow{k_i} \underbrace{k_i}_{s} \xrightarrow{u_c} \underbrace{Plant}_{r} \xrightarrow{\Omega_r}_{r}$ 

Fig. 2 Regulator IP

The expression of the output  $\Omega_r$  is:

$$\Omega_{\rm r} = \frac{kk_{\rm p}k_{\rm i}\Omega_{\rm ref} - s\tau_{\rm L}}{Js^2 + (B + k_{\rm p}k)s + kk_{\rm p}k_{\rm i}}$$
(37)

The desired torque can be also defined by [3].

$$\tau^{\bullet} = J\dot{\omega}_{m}^{\bullet} - z + \tau_{L}$$

$$\dot{z} = -az + b(\omega_{m} - \omega_{m}^{\bullet}), \ a, b > 0$$
(38)

#### 3.3.3. Passivity based control with oriented flux

By the application of non linear feedback state [3] which allows to maintain the reaction of the flux of inductance in quadrature with the rotor flux. The orientation control of flux will impose a null current  $i_d$  along the direct axis. The behavior of the PMSM is similar to that of CC motor; thus the control present linear feedback expressed by the relation:

$$V_{d} = -L_{a}p\omega_{m}i_{a}$$
(39)

The application of the (39) of the model (15)-(17) impose a null  $i_d$  current; we obtain the simplified model:

$$L_{q}\frac{di_{q}}{dt} + R_{a}i_{q} = V_{q}$$
(40)

$$J\dot{\omega}_{\rm m} + B\omega_{\rm m} = \tau - \tau_{\rm L} \tag{41}$$

$$\tau = -\frac{3}{2} p \phi_{\rm f} i_{\rm q} \tag{42}$$

If the quadratic flux  $\psi_q$  is taken as a state variable and the rotor speed  $\omega_m$ , from equations (20) and (40)-(42) we obtain the following model:

$$\dot{\psi}_{q} - p\omega_{m}\phi_{f} = V_{q} - R_{a}i_{q}$$
(43)

$$J\dot{\omega}_{\rm m} + B\omega_{\rm m} = \tau - \tau_{\rm L} \tag{44}$$

$$\tau = -\frac{3}{2}p\phi_{\rm f}i_{\rm q} \tag{45}$$

The control of PSMS in current using the loop PI of current  $i_q$  given by relation (23) we obtain the model

$$\dot{\psi}_{q} - p\omega_{m}\phi_{f} = -R_{a}\dot{i}_{q}^{\bullet}$$
(46)

$$J\dot{\omega}_{\rm m} + B\omega_{\rm m} = \tau - \tau_{\rm L} \tag{47}$$

$$\tau = -\frac{3}{2} p \phi_f i_q^{\bullet} \tag{48}$$

The equations (44)-(46) represent a mono-input model which will be used for the synthesis of  $i_q^{\bullet}$  by passivity. Following the same steps (28)-(38) we obtain the following –PBC.

$$\mathbf{i}_{\mathbf{q}}^{\bullet} = \frac{1}{\mathbf{R}_{\mathbf{a}}} \left( \left( -\psi_{\mathbf{q}}^{\bullet} + \mathbf{p}\omega_{\mathbf{m}}\phi_{\mathbf{f}} \right) + \mathbf{k}_{\mathbf{f}}\mathbf{e}_{\mathbf{f}\mathbf{q}} \right)$$
(49)

with  $\psi_q^{\bullet}$  given by (36) and the desired torque  $\tau^{\bullet}$  by regulator IP. Or relation (38) for speed control.



Fig. 3 General diagram of the PBC without mechanical sensor



Fig. 4 Response of the passivity based control on the inversion of the sense of rotation and application of the couple of load to t=0.4s operation without converter.



Fig. 5 Test of robustness for a variation of the parameters of the motor for:

a-	$R=2R_n, L=0.5L_n.$
b-	$R=2R_n, L=1.5L_n.$

c-  $\phi_f = 0.5 \phi_{fn}$ 

### 4. SLIDING MODE OBSERVER [5]

The  $\alpha$ - $\beta$  model for the PMSM in the stationary reference frame is characterized by (50):

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_{s} = \mathbf{A}\mathbf{i}_{s} + \mathbf{B}_{\mathrm{L}}\mathbf{v}_{s} + \mathbf{B}_{\mathrm{L}}\mathbf{E}_{s}$$
(50)

where

$$\mathbf{i}_{s} = (\mathbf{i}_{\alpha} \ \mathbf{i}_{\beta})^{\mathrm{T}} \quad : \text{ stator } \alpha \text{ - and } \beta \text{ -axes currents}$$
$$\mathbf{v}_{s} = (\mathbf{v}_{\alpha} \ \mathbf{v}_{\beta})^{\mathrm{T}} : \text{ stator } \alpha \text{ - and } \beta \text{ -axes voltages}$$
$$\mathbf{E}_{s} = \begin{pmatrix} \mathbf{E}_{\alpha} = \varphi_{\alpha} \omega_{r} = -\phi_{f} \omega_{r} \sin \theta \\ \mathbf{E}_{\beta} = \varphi_{\beta} \omega_{r} = \phi_{f} \omega_{r} \cos \theta_{r} \end{pmatrix} : \text{ induced voltage}$$
$$\begin{bmatrix} \mathbf{I}_{\alpha} & \mathbf{0} \end{bmatrix}$$

A = (-R<sub>a</sub> / L).I; B<sub>L</sub> = (1/L).I; I =  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 

Considering the PMSM, the rotor speed and the stator resistance are variable, which are to be estimated. From (50), the sliding observer is made as the following structure.

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{i}}_{s} = \hat{\mathbf{A}}\hat{\mathbf{i}}_{s} + \mathbf{B}_{\mathrm{L}}\mathbf{v}_{s} + \mathbf{B}_{\mathrm{L}}\hat{\mathbf{E}}_{s} - \mathbf{K}$$
(51)

where:  

$$\begin{array}{l}
\mathbf{A} = (-\hat{\mathbf{R}}_{a} / \mathbf{L}).\mathbf{I} \\
\mathbf{K} = \zeta \mathbf{S} + \Psi
\end{array}$$
: observer input
$$\zeta = \begin{bmatrix} \zeta_{11} & \mathbf{0} \\ \mathbf{0} & \zeta_{22} \end{bmatrix}; \ \vartheta = \begin{bmatrix} \vartheta_{1} \\ \vartheta_{2} \end{bmatrix} : \text{ observer gain}$$

$$\uparrow: \text{ estimated values.}$$

The sliding hyper plane is defined upon the stator current errors.

$$\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2)^{\mathrm{T}} = \hat{\mathbf{i}}_{\mathrm{s}} - \mathbf{i}_{\mathrm{s}} = \mathbf{e}_{\mathrm{s}}$$
(52)

The estimation error dynamic is given by the following equation.

$$\dot{\mathbf{S}} = \frac{\mathbf{d}}{\mathbf{dt}}(\hat{\mathbf{i}}_{s} - \mathbf{i}_{s}) = (\hat{\mathbf{A}}\hat{\mathbf{i}}_{s} - \mathbf{A}\mathbf{i}_{s}) + \mathbf{B}_{L}(\hat{\mathbf{E}}_{s} - \mathbf{E}_{s}) - \mathbf{K}$$
(53)

### 4.1. Estimator of speed and stator resistance

In order to estimate rotor speed and stator resistance, a Lyapunov's function V is used.

The Lyapunov's function is chosen as

$$V = \frac{1}{2}S^{T}.S + \frac{(\hat{\omega}_{r} - \omega_{r})^{2}}{2} + \frac{(\hat{R}_{s} - R_{s})^{2}}{2}$$
(54)

Under the assumption that the rotor speed is constant within one estimation period, derivative of the Lyapunov's function becomes:

$$\dot{\mathbf{V}} = \dot{\mathbf{S}}^{\mathrm{T}} \cdot \mathbf{S} + (\hat{\boldsymbol{\omega}}_{\mathrm{r}} - \boldsymbol{\omega}_{\mathrm{r}}) \dot{\hat{\boldsymbol{\omega}}}_{\mathrm{r}} + (\hat{\mathbf{R}}_{\mathrm{a}} - \mathbf{R}_{\mathrm{a}}) \cdot \hat{\mathbf{R}}_{\mathrm{a}}$$
(55)

.

Substituting (53) into (55), then following equation is obtained

$$\dot{\mathbf{V}} = \mathbf{S}^{\mathrm{T}} \left[ (\hat{\mathbf{A}} - \mathbf{A}) \cdot \hat{\mathbf{i}}_{\mathrm{s}} + \mathbf{A} \cdot (\hat{\mathbf{i}}_{\mathrm{s}} - \mathbf{i}_{\mathrm{s}}) + \mathbf{B}_{\mathrm{L}} \cdot (\hat{\mathbf{E}}_{\mathrm{s}} - \mathbf{E}_{\mathrm{s}}) - \mathbf{K} \right]$$
$$+ \Delta \omega_{\mathrm{r}} \dot{\hat{\omega}}_{\mathrm{r}} + \Delta \mathbf{R}_{\mathrm{a}} \dot{\hat{\mathbf{R}}}_{\mathrm{a}}$$
(56)

where,  $\Delta \omega_r = \hat{\omega}_r - \omega_r$ ,  $\Delta R_a = \hat{R}_a - R_a$ . According to the Lyapunov's stability theory,  $(\dot{V} < 0)$  must be obeyed to guarantee that the observer is stable. In order to drive the system to be convergent, let:

$$S^{T}.[(\hat{A} - A).\hat{i}_{s}] + S^{T}[B_{L}.(\hat{E}_{s} - E_{s})] + \Delta \omega_{r}\dot{\hat{\omega}}_{r} + \Delta R_{a}\dot{\hat{R}}_{a} = 0$$
(57)

$$\mathbf{S}^{\mathrm{T}}[\mathbf{A}.(\hat{\mathbf{i}}_{s}-\mathbf{i}_{s})-\mathbf{K}]<\mathbf{0}$$
(58)

From (57), estimation algorithms of the rotor speed and the stator resistance may be derived. Also, the observer input must be chosen to satisfy the inequality (58). Rearranging (57) appropriately then two novel equations are obtained as

$$\mathbf{S}^{\mathrm{T}}[(\hat{\mathbf{A}} - \mathbf{A}) \cdot \hat{\mathbf{i}}_{s}] + \Delta \mathbf{R}_{a} \hat{\mathbf{R}}_{a} = \mathbf{0}$$
(59)

$$\mathbf{S}^{\mathrm{T}}[\mathbf{B}_{\mathrm{L}}.(\hat{\mathbf{E}}_{\mathrm{s}}-\mathbf{E}_{\mathrm{s}})] + \Delta \omega_{\mathrm{r}} \dot{\hat{\boldsymbol{\omega}}}_{\mathrm{r}} = \mathbf{0}$$
(60)

From (59), the stator resistance estimator may be derived as

$$\dot{\hat{\mathbf{R}}}_{\mathbf{a}} = \frac{1}{\mathbf{L}_{\mathbf{s}}} (\mathbf{s}_1 \cdot \hat{\mathbf{i}}_{\alpha} + \mathbf{s}_2 \cdot \hat{\mathbf{i}}_{\beta})$$
(61)

Also, from (59), the speed estimator may be derived

as: 
$$\hat{\omega}_{\mathbf{r}} = \frac{\Phi_{\mathbf{f}}}{\mathbf{L}_{\mathbf{s}}} \mathbf{S}^{\mathrm{T}} \begin{bmatrix} \sin \hat{\theta}_{\mathbf{r}} \\ -\cos \hat{\theta}_{\mathbf{r}} \end{bmatrix} = \frac{\Phi_{\mathbf{f}}}{\mathbf{L}_{\mathbf{s}}} (\mathbf{s}_{1} \cdot \sin \hat{\theta}_{\mathbf{r}} - \mathbf{s}_{2} \cdot \cos \hat{\theta}_{\mathbf{r}})$$
(62)

The estimated rotor position is obtained by integrating the rotor speed.

# 4.2. Design of observer gain

\_ .

In order to guarantee the derivative of the Lyapunov function  $(\dot{V} < 0)$ , the observer gains and for observer input must be chosen the satisfy inequality (58)

ISSN 1335-8243 © 2009 FEI TUKE

$$\mathbf{S}^{\mathrm{T}}[\mathbf{A}.(\hat{\mathbf{i}}_{s} - \mathbf{i}_{s}) - \zeta \mathbf{S} - \vartheta] < \mathbf{0}$$
(63)

The sufficient conditions for satisfying the inequality (63) are:

$$\boldsymbol{\zeta} > \mathbf{A} \qquad \mathbf{S}^{\mathrm{T}}, \boldsymbol{\vartheta} > \mathbf{0} \tag{64}$$

The condition for satisfying inequality (64) can be respectively derived as

$$\zeta_{11}, \quad \zeta_{22} > \frac{\hat{R}_a}{L}; \vartheta i = \begin{cases} \alpha_i & \text{si } s_i > 0\\ -\beta_i & \text{si } s_i < 0 \end{cases}$$

where,  $\alpha_i$ ,  $\beta_i$ : positive constant.

### 4.3. Results of simulation with observer speed



Fig. 6 Response of the passivity based control on inversion of the sense of rotation and application of the couple of load from t=0.4s to 0.6s operation without mechanical sensor.

# 5. VALIDATION BY NUMERICAL SIMULATION

The performance of the proposed control has been tested by numerical simulation for the motor whose parameters are given in appendix.

The Fig. 4 shows the speed response from 0-100rd/s following by sudden loading (application of a nominal torque decoupling of PBC; the disturbance rejection is satisfactory and the speed response is without overtaking and no static error.

The Fig. 5 shows the results with the variation of the parameters of the motor. The speed response from  $\pm$  100 rd/s is made with a rapid rejection of perturbation and therefore the system is insensitive to parametric variations. Thus the response of the system is convenient and is robust.

The simulation had shown that the control system of PBC proposed gives good performance the tracking of the speed is robust with respect to the disturbance represented by the nominal loading torque and the parametric variations (electrical and mechanical).

One will present in what follows results of simulation of a training with base of one PSMS fed with a voltage converter without sensor of speed

The Fig. 6 presents the evolution of the estimated speed with the PSMS with introduction of a load of t = 0.4 - 0.6 and change of the sense of rotation in t=2s. One notices after comparison that these results are practically identical with those found in the case of use of the mechanical sensors; one can so assert that the SMO allowed us well to estimate speed of rotation. One notices besides in the Fig. 6 that control without mechanical sensor presents a good robustness of measure to the low speeds.

### 6. CONCLUSION

We have presented in the present article the control based on passivity applied to the association of three level–PMSM. The objective of speed tracking disturbance rejections is acceptable. The decoupling is maintained even in the case of load variation.

A good choice of injection and damping coefficient to the current control is allowed to have a very good performance with robustness and can be applied to applications of high performance such as machine tools and robots. To show the validity of control suggested without mechanical sensor, we thus implemented and validated by simulation an observer at sliding mode of complete order adaptive. It advantageously makes it possible to estimate speed and to compensate for the parametric variations.

### REFERENCES

- P.C. Krause, O. Wasynczuk et S.D. Sudhoff: "Analysis of electrical machines", IEEE Press, 1994.
- [2] Jean Matthieu B.: Thèse de Doctorat de L'INPG " Contribution à la commande de systèmes mécaniques non-réguliers". Octobre 2004.
- [3] A.Y. Achour, Commandes avec observateurs basés sur la passivité: application à la machine synchrone à aimant permanents, Thèse de Magister, EMP, Alger, Algérie, 1999.
- [4] Reda Boukezzoula : Thèse de Doctorat de l'université de Savoie. "Commande d'une classe de systèmes non linéaires". Mars 2000.
- [5] Yoon-Seok Han, Jung-Soo Choi, and Young-Seok Kim "Sensorless PMSM Drive with a Sliding Mode Control Based Adaptive Speed and Stator Resistance Estimator". IEEE TRANS ON MAGNETICS, VOL. 36, NO. 5, SEPT 2000.

### **APPENDIX: PARAMETERS OF MOTOR**

(In smooth poles  $L_d=L_q=L$ )

$$\begin{split} P_{N} = &600W; \ \Omega = &150 \ rd/s; \ p = 1; \ L = &20.5 \ mH; \\ R_{a} = &1.55\Omega; \ \varphi_{f} = &0.22N.m/A; \ J = &2.2x10^{-3}Kgm^{2}; \\ B = &2.2x10^{-3} \ Nms/rad; \ Vmax = &300V; \ Imax = &20A. \end{split}$$



Fig. 7 Observer with adaptive sliding mode

Received May 13, 2008, accepted January 29, 2009

# BIOGRAPHIES

**Baghdad Belabbes** was born in Sidi belabbes Algeria. He received his BS degree in electrical engineering from the Electrical Engineering Institute of Annaba 1976 and the MS degree from the Electrical Engineering Institute of Sidi Bel-Abbes University in 2002 where he is currently Professor of electrical engineering. He is a member in Intelligent Control Electrical Power System Laboratory (ICEPS).

**Mohamed Karim Fellah** was born in Oran, Algeria, in 1963. He received the Eng. degree in Electrical Engineering from University of Sciences and Technology, Oran, Algeria, in 1986, and The Ph.D. degree from National Polytechnic Institute of Lorraine (Nancy, France) in 1991. Since 1992, he is Professor at the University of Sidi-bel-Abbes (Algeria) and Director of the Intelligent Control and Electrical Power Systems Laboratory at this

University. His current research interest includes power electronics, HVDC links, and drives.

Abdelkader Lousdad was born in at Sidi Bel-Abbes. Graduated from the University of Cincinnati USA in 1979 in Mechanical Engineering. Holds a Master in Industrial Management of Enterprises in 1989 and a Magister in 1992 in Structure/ Robotics from the University of Sidi Bel-Abbes. He obtained his Doctorare degree in 2008. Former Mechanical and Industrial engineer in fields of production, product assembly, industrial maintenance, quality control, Methods and R&D, product design and manufacturing, and related technologies to Environmental Protection, Industrial and Technical Management Auditing. Has been Central Technical and Research Coordinator, then a Supervisor of Industrial Engineering Research and Design Center. Currently he is Assistant Professor at the Mechanical Engineering Department at the University of Sidi-Bel-Abbes. Research interest: Research focus is on applying System Concepts and Optimization, Design and Prototyping Techniques to Engineering Sciences and Technologies and related technologies and topics to Environmental Protection. He is a member of University Research Laboratory in "Mechanics of Materials and reactive systems."

- Co Worker in: "Intelligent Control Electronic Power System Laboratory (I.C.E.P.S)"

- Co Worker in: "IRECOM Laboratory"

**Meroufel Abdelkader** was born in 1954 in Ain El Berd, Sidi-bel-Abbes , Algeria. He received his BS degree in electrical engineering from the Electrical Engineering Institute of Oran 1980 and The Ph.D. degree from the Electrical Engineering Institute of Sidi Bel-Abbes University in 2004 where he is currently Professor of electrical engineering. He is a member in Intelligent Control Electrical Power System Laboratory (ICEPS).His current research interest includes power electronics and electrical control.

**Mohamed Abid** received the B.S., M.S., and Ph.D degrees in electrical engineering from University of Sidi Bel Abbes, Algeria, in 1990, 1997, and 2005, respectively. He is currently professor of electrical engineering at University of Sidi Bel Abbes, Algeria. His mains areas of research interests are modelling and simulation of electric machines, sliding mode in continuous systems, control of electric drives.