# CAPACITANCE CALCULATION USING EEM 

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#### Abstract

Sometimes physical systems can be so complex that analytical solutions for potentials or electric fields strength of those problems are difficult or even impossible to be found. Fortunately, with the application of the computer, it is now possible to use simple numerical approximation methods to solve more complex problems in a matter of a few minutes. For determination an unknown capacitance can be used different numerical methods. In this article, the Equivalent Electrodes Method (EEM) [9] is used for calculation of system capacitance. A sphere and thin ring form this system. In paper [5], the authors called that system "Saturn" capacitor. In this article, the capacitance of this system, when the capacitor ring has finite or negligible thickness is calculated. Also, the ring cross-section can have different shapes. The capacitance calculation has been done for several ring cross-section shapes. The Equivalent Electrodes Method, Point-matching Method and Image theorem are used for calculation. The convergence of the results will be shown in the tables, and the capacitance values for different values of parameters will be shown graphically. Also, the obtained results for capacitance for different ring cross-section shapes will be shown in the tables.

In last few years there has been developed a large number of software packages for solving problems in Electromagnetics. They make calculations easier and it is also a good way to confirm the results obtained by some analytical or numerical method. In this article, the obtained EEM results will be compared with FEMM software [10] results.


Keywords: Capacitance calculation, Equivalent Electrodes Method (EEM), Point-matching Method (PMM), Image theorem, Finite Element Method (FEM).

## 1. INTRODUCTION

A sphere of radius $a$ and a thin ring with negligible or finite thickness, having inner radius $b$ and exterior radius $c$, form so-called "Saturn" capacitor [5]. The ring is at the potential $U$, the sphere is at the potential $V$ and placed at the height $h$, Fig. 1 .


Fig. 1 "Saturn" capacitor.
The most common method for electrophysiological investigation for ion channel proteins is the twomicroelectrode voltage clamp technique [3], Fig. 2. A spherical electrode and a thin ring electrode form one part of that system i.e. "Saturn" capacitor.

In references [2, 5, 6] a capacitance of the "Saturn" capacitor is considered. In papers [2,5] a thin ring with a negligible thickness has been observed. In the paper [6] the ring has had a finite thickness. In that case, the ring shape cross-section was rectangular. An influence of different ring cross-section shapes is considered in the paper [7].

In this paper will be presented all obtained results and those results will be compared with the Finite Element Method (FEM) results.


Fig. 2 Two-microelectrode voltage clamp system.

## 2. EQUIVALENT ELECTRODES METHOD (EEM)

This method has been developed at the Faculty of Electronic Engineering in Niš - Department of Theoretical Electrical Engineering and it belongs to the group of Semi Numerical Methods. The first very good results were obtained in [8], when this method is used for calculating the equivalent radius of uniform antennas. A basic idea of this method is replacing an electrostatic system by a finite system of equivalent electrodes (EEs) [9].

The equivalent electrodes of different shapes can be used depending on the problem geometry. The flat or oval strips (for plan-parallel problems), spherical bodies (for three-dimensional problems) or toroidal electrodes (for systems with axial symmetry) can be commonly used.

The equivalent electrodes potential should be equal to the real electrode potential. The system of linear equations is formed using this boundary condition, with equivalent
electrodes charges as unknown values. After solving this system, the unknown charges of EEs can be determined. Using standard formulas the potential, the electric field strength and the capacitance of the system can be computed.

The EEM has a wide range of applications [9]. This method was applied in computation of electrostatic fields, in theory of low-frequency grounding systems, in the magneto static field and heat flow problems solving, transmission line analysis, etc.

In [10], this method is applied for electric field and potential determination at the coaxial cable terminations and joints. Toroidal electrodes are used as EEs. In [11] the EEs are small spherical bodies used to determine the atmospheric electric field distribution in the surroundings of the vehicles. A potential distribution in vicinity of biological bodies exposed to ELF electric field is determined in [12] using the EEs of identical shape as in [11]. Flat or oval strips elements of large length and neglected width can be used as the EEs for electromagnetic field analysis which slit coaxial lines produce in tunnels and in bridges with one or double track [13].

## 3. EEM APPLICATION

### 3.1. Ring with negligible thickness

For determination the capacitance of the system from Fig. 1, when the ring has a negligible thickness $(\delta \rightarrow 0)$, the ring is divided in $N$ strips, Fig. 3 [1].


Fig. 3 EEM application.

The radius of $n$-th strip is

$$
\begin{equation*}
r_{n}=0.5 b\left[\left(\frac{c}{b}\right)^{n / N}+\left(\frac{c}{b}\right)^{(n-1) / N}\right], \tag{1}
\end{equation*}
$$

and its width is
$\Delta l_{n}=b\left[\left(\frac{c}{b}\right)^{n / N}-\left(\frac{c}{b}\right)^{(n-1) / N}\right]$.

Each of the formed strips can be replaced by equivalent loops, having radius $r_{n}$, with the circular crosssection of radius $a_{\mathrm{e} n}=\Delta l_{n} / 4$.

The sphere and $N$ loops form the system. Applying the image theorem in the sphere mirror, the equivalent system is formed. The charges of the loops, their images in the sphere mirror and one point charge placed in the centre of the sphere form this system, Fig. 4.

The images are loops, too, with radii
$r_{n}^{\prime}=r_{n}\left(\frac{a}{d_{n}}\right)^{2}$,
and
$Q_{n}^{\prime}=-\frac{a}{d_{n}} Q_{n}$
is charge of the $n$-th loop image.


Fig. 4 Equivalent system.

The ring and the sphere form a capacitor, so their charges must be equal, but of the opposite sign. Because of that, the total sphere charge must be $-Q$ [14]. One part of this charge is divided into image charges, obtained using image theorem. The influence of these charges can be presented with one equivalent loop, and the other charges
$\Delta q_{n}=-Q_{n}-Q_{n}^{\prime}=-\left(1-\frac{a}{d_{n}}\right) Q_{n}$
are uniformly placed at the sphere surface, so their influence can be presented with point charge placed in the centre of the sphere. Its intensity is
$\Delta q=\sum_{n=1}^{N} \Delta q_{n}$.

The potential in point $\mathrm{M}(r, z)$ is

$$
\varphi=\sum_{n=1}^{N} \frac{1}{2 \pi^{2} \varepsilon}\left[Q_{n} \frac{\mathrm{~K}\left(\frac{\pi}{2}, k_{n}\right)}{\sqrt{\left(r+r_{n}\right)^{2}+(z-h)^{2}}}+\right.
$$

$$
+Q_{n}^{\prime} \frac{\mathrm{K}\left(\frac{\pi}{2}, k_{n}^{\prime}\right)}{\sqrt{\left(r+r_{n}^{\prime}\right)^{2}+\left(z-h\left(\frac{a}{d_{n}}\right)^{2}\right)^{2}}}+
$$

$$
\begin{equation*}
\left.+\frac{\pi}{2} \frac{\Delta q_{n}}{\sqrt{r^{2}+z^{2}}}\right] \tag{7}
\end{equation*}
$$

where $\mathrm{K}\left(\frac{\pi}{2}, k_{n}\right)$ and $\mathrm{K}\left(\frac{\pi}{2}, k_{n}^{\prime}\right)$ are complete elliptic integrals of the first kind, with modules

$$
k_{n}^{2}=\frac{4 r r_{n}}{\left(r+r_{n}\right)^{2}+(z-h)^{2}}
$$

and

$$
k_{n}^{\prime 2}=\frac{4 r r_{n}^{\prime}}{\left(r+r_{n}^{\prime}\right)^{2}+\left(z-h\left(\frac{a}{d_{n}}\right)^{2}\right)^{2}}
$$

When the potential in some point $\mathrm{M}(r, z)$ is matched in $N$ matching points placed at the electrodes surfaces, the system of linear equations is formed:

$$
\begin{align*}
& U=\sum_{n=1}^{N} \frac{Q_{n}}{2 \pi^{2} \varepsilon}\left[\frac{\mathrm{~K}\left(\frac{\pi}{2}, k_{m n}\right)}{\sqrt{\left(r_{m}+r_{n}\right)^{2}+\delta_{m n} a_{\mathrm{e} m}^{2}}}-\right. \\
& -\frac{a}{d_{n}} \frac{\mathrm{~K}\left(\frac{\pi}{2}, k_{m n}^{\prime}\right)}{\left(r_{m}+r_{n}^{\prime}\right)^{2}+h^{2}\left(1-\left(\frac{a}{d_{n}}\right)^{2}\right)^{2}}- \\
& \left.-\frac{\pi}{2 \sqrt{r_{m}^{2}+h^{2}}}\left(1-\frac{a}{d_{n}}\right)\right], \quad m=1,2, \ldots, N \tag{8}
\end{align*}
$$

where $\delta_{m n}$ is Kronecker symbol.
$k_{m n}^{2}=\frac{4 r_{m} r_{n}}{\left(r_{m}+r_{n}\right)^{2}+\delta_{m n} a_{\mathrm{e} m}^{2}}$ and

$$
k_{m n}^{\prime 2}=\frac{4 r_{m} r_{n}^{\prime}}{\left(r_{m}+r_{n}^{\prime}\right)^{2}+h^{2}\left(1-\left(\frac{a}{d_{n}}\right)^{2}\right)^{2}}
$$

are modules of the elliptic integrals.
After solving system (8) the capacitance can be calculated as
$C=\frac{Q}{U-V}$
where
$Q=\sum_{n=1}^{N} Q_{n}$
is the total charge of the ring electrode.
The sphere potential, denoted with $V$, derives from the charge $\Delta q$, placed in the centre of the sphere. Using condition that the sphere is equipotential this potential can be determined:
$\frac{V}{U}=-\frac{\pi}{2} \sum_{n=1}^{N}\left(1-\frac{a}{r_{n}}\right) \frac{Q_{n}}{2 \pi^{2} \varepsilon a}$.

### 3.2. Ring with finite thickness

A problem is more complex when the ring has finite thickness. In that case, all four sides of the ring crosssection should be divided in strips.

Upper and bottom sides of the ring cross-section are divided in the same way as in the case of the ring with the negligible thickness. But, number of the equivalent electrodes on the upper and bottom ringsides can be different. So, the ring is divided in $N_{j}$ strips using expressions (1) and (2) [14], where $j=1,2$.

Indexes 1 and 2 correspond to the strips on the upper and bottom ringside, respectively.

Depends on the ring cross-section shape, inner and exterior sides can be divided in different ways. If the ring cross-section shape is rectangular, Fig. 5, inner and exterior sides, with radii $r=b$ and $r=c$, have been divided in $N_{3}=N_{4}$ ring strips, with width
$\Delta l_{k n}=\frac{\delta}{N_{k}}$,
( $n=1, \ldots, N_{3}$ ), placed at positions
$h_{k n}=h-\frac{\delta}{2}+(2 n-1) \frac{\Delta l_{k n}}{2}$.
where $k=3,4$. Indexes 3 and 4 correspond to inner ( $r_{3 n}=b$ ) and exterior ( $r_{4 n}=c$ ) ringside, respectively. Each of the formed strips can be replaced by equivalent loops, having radius $r_{k n}$ with circular cross-section of radius $a_{\mathrm{e} k n}=\Delta l_{k n} / 4, k=1,2,3,4$.


Fig. 5 Rectangular shape of the ring cross-section.
If the inner and exterior sides of ring cross-section are half circular, with radius $\delta / 2$, Fig. 6, they will be divided in $N_{3}=N_{4}$ strips with width $\delta \Delta \theta / 2$, where
$\Delta \theta=\frac{\pi}{N_{k}}$
where $k=3,4$. Indexes 3 and 4 correspond to inner and exterior sides of ring cross-section, respectively.


Fig. 6 Shape of the ring cross-section.
Each of the formed strips can be replaced by equivalent loops with circular cross-section of radius
$a_{\mathrm{e} k n}=\frac{\delta}{2} \sin \left(\frac{\Delta \theta}{4}\right)$.
The radii of formed strips are
$r_{k n}=b-\frac{\delta}{2} \cos \theta_{k n}$ and
$r_{k n}=c+\frac{\delta}{2} \cos \theta_{k n}$,
for inner and exterior sides of ring cross-section, respectively. The normal distance of these strips from the sphere centre is
$h_{k n}=h-\frac{\delta}{2} \sin \theta_{k n}$,
where
$\theta_{k n}=\frac{\pi}{2 N_{k}}\left(2 n-1-N_{k}\right)$,
$\left(n=1,2, \ldots, N_{k}\right.$ and $\left.k=3,4\right)$.
Application of previous expressions depends on the ring cross-section shape. If the exterior side of ring crosssection is flat, the strips will be formed using the expression (12). If the inner side is half circular, the strips will have the positions described using the expressions (15) and (17).

For any ring cross-section shape, the sphere and $N$ loops, where $N=N_{1}+N_{2}+N_{3}+N_{4}$, form the system.

Applying the image theorem in the sphere mirror, the equivalent system is formed as in chapter 3.1. The charges of the loops, their images in the sphere mirror and one point charge placed in the centre of the sphere form this system. In this case only the number of equivalent electrodes is different. Also, the number of images is bigger.

The potential at point $\mathrm{M}(r, z)$ is
$\varphi=\sum_{i=1}^{4} \sum_{n=1}^{N_{i}} \frac{1}{2 \pi^{2} \varepsilon}\left[Q_{i n} \frac{\mathrm{~K}\left(\frac{\pi}{2}, k_{i n}\right)}{\sqrt{\left(r+r_{i n}\right)^{2}+z^{2}}}+\right.$
$\left.+Q_{i n}^{\prime} \frac{\mathrm{K}\left(\frac{\pi}{2}, k_{i n}^{\prime}\right)}{\sqrt{\left(r+r_{i n}^{\prime}\right)^{2}+z^{2}}}+\frac{\pi}{2} \frac{\Delta q_{i n}}{\sqrt{r^{2}+z^{2}}}\right]$,
where $\mathrm{K}\left(\frac{\pi}{2}, k_{\text {in }}\right)$ and $\mathrm{K}\left(\frac{\pi}{2}, k_{\text {in }}^{\prime}\right)$ are complete elliptic integrals of the first kind, with modulus
$k_{i n}^{2}=\frac{4 r r_{i n}}{\left(r+r_{i n}\right)^{2}+z^{2}}$ and $k_{i n}^{\prime 2}=\frac{4 r r_{i n}^{\prime}}{\left(r+r_{i n}^{\prime}\right)^{2}+z^{2}}$,
and $Q_{i n}^{\prime}=-\frac{a}{r_{i n}} Q_{i n}$ is charge of the $n$-th loop image ( $i=1,2,3,4$ ).

With $\Delta q_{i n}(i=1,2,3,4)$ the point charges placed in the centre of the sphere are denoted. Parameter $N_{i}$ ( $i=1,2,3,4$ ) corresponds to the equivalent electrodes number of each ringside.

The unknown charges $Q_{i n}$ can be determined when the potential (19) is matched in $N$ matching points placed at the electrodes surfaces.

After solving formed system of linear equations the capacitance can be calculated.

## 4. NUMERICAL RESULTS

In Table 1 and Table 2, the capacitance values, for different number of equivalent electrodes (EEs) and different values of the parameters, are shown. Those results are presented for the ring with the negligible thickness.

Table 1 Normalized capacitance for different number of EEs, for $b / a=1.5, h / a=1.0$ and

|  | $c / a=2.5$ | $c / a=8.0$ |
| :---: | :---: | :---: |
| $N$ | $C / 2 \pi^{2} \varepsilon a$ | $C / 2 \pi^{2} \varepsilon a$ |
| 20 | 0.822387810001 | 0.755306923094 |
| 30 | 0.821834725333 | 0.754993298392 |
| 50 | 0.821313034730 | 0.754783322516 |
| 100 | 0.820842802590 | 0.754667353520 |
| 150 | 0.820659718595 | 0.754642755134 |
| 200 | 0.820560021664 | 0.754663485378 |

The obtained results have shown that small number of the equivalent electrodes, the good convergence of the results.

Table 2 Normalized capacitance for different number of EEs, for $b / a=1.5, h / a=8.0$ and

|  | $c / a=2.5$ | $c / a=8.0$ |
| :---: | :---: | :---: |
| $N$ | $C / 2 \pi^{2} \varepsilon a$ | $C / 2 \pi^{2} \varepsilon a$ |
| 20 | 0.452348182622 | 0.637047922345 |
| 30 | 0.452225635291 | 0.636923436253 |
| 50 | 0.452086095869 | 0.636789099615 |
| 100 | 0.451940052984 | 0.636650600848 |
| 150 | 0.451877563907 | 0.636591167078 |
| 200 | 0.451842043759 | 0.636557226094 |

In Figs. 7 and 8, the capacitance values for different parameters are shown. The number of the EEs is $N=100$. These results are presented for the negligible ring thickness.

From Fig. 7, it is evident that when distance between the ring and the sphere increases and the ring width decreases, the capacitance values stream to identical value. That is because the sphere doesn't "see" the ring width. When the distance between the sphere and the ring increases, the capacitance is almost constant. That can be seen from Fig. 8.

In Table 3 and Table 4, the capacitance values, for different number of the equivalent electrodes and different values of parameters, are shown. The ring has a finite thickness and the ring cross-section shape is rectangular as in Fig. 5. From these tables the good convergence of the results can be noticed.

The ring thickness is given using the parameter $\Delta$, where

$$
\begin{equation*}
\Delta=\frac{\delta}{c-b} \tag{20}
\end{equation*}
$$



Fig. 7 Capacitance versus parameter $b / a$.



Fig. 8 Capacitance versus parameter $h / a$.

Table 3 Capacitance for different number of EEs, for $b / a=2.0, c / a=3.0$ and $h / a=0.0$.

| $\Delta=0.01$ |  |  | $\Delta=0.1$ |
| :---: | :---: | :---: | :---: |
| $N$ | $C / 2 \pi^{2} \varepsilon a$ | $N$ | $C / 2 \pi^{2} \varepsilon a$ |
| 20 | 0.8517249114 | 20 | 0.8906905258 |
| 30 | 0.8591924056 | 30 | 0.8895970007 |
| 50 | 0.8615284292 | 50 | 0.8886667993 |
| 80 | 0.8617913541 | 80 | 0.8882241875 |
| 100 | 0.8617266748 | 100 | 0.8880980130 |
| 150 | 0.8615377676 | 150 | 0.8878836563 |
| 200 | 0.8614092168 | 200 | 0.8877595248 |

In Tables 5-8, the capacitance values for different shapes of ring cross-section and different values of parameters are shown. The rectangular cross-section is "Shape 1 ". For "Shape 2 " the inner side is half circular and exterior side of ring cross-section is flat. "Shape 3" corresponds to the shape presented in Fig.6.

Table 4 Capacitance for different number of EEs, for $b / a=2.0, c / a=3.0$ and $h / a=1.0$.

| $\Delta=0.01$ |  | $\Delta=0.1$ |  |
| :---: | :---: | :---: | :---: |
| $N$ | $C / 2 \pi^{2} \varepsilon a$ | $N$ | $C / 2 \pi^{2} \varepsilon a$ |
| 20 | 0.7847170894 | 20 | 0.8165307862 |
| 30 | 0.7908987714 | 30 | 0.8154089370 |
| 50 | 0.7926265740 | 50 | 0.8145657420 |
| 80 | 0.7931838896 | 80 | 0.8141692807 |
| 100 | 0.7931068251 | 100 | 0.8140549214 |
| 150 | 0.7929422077 | 150 | 0.8138765474 |
| 200 | 0.7928340829 | 200 | 0.8137751744 |

Table 5 Capacitance for different ring cross-section shapes, for $b / a=2.0, c / a=3.0$ and $h / a=0.0$.

| $N=200$ | $C / 2 \pi^{2} \varepsilon a$ |  |  |
| :---: | :---: | :---: | :---: |
| $\Delta$ | Shape 1 | Shape 2 | Shape 3 |
| 0.01 | 0.86140922 | 0.86251650 | 0.86284000 |
| 0.05 | 0.87442003 | 0.88019209 | 0.88164570 |
| 0.10 | 0.88775952 | 0.89935978 | 0.90184113 |
| 0.15 | 0.89951629 | 0.91740597 | 0.92068939 |
| 0.20 | 0.91024679 | 0.93494090 | 0.93885055 |

From these tables it can be found that if the distance between the sphere and the ring increases, the difference between capacitance results for different shapes of ring cross-section decreases.

Also, the bigger ring thickness, the bigger capacitance value is obtained. The smallest capacitance values are for the "Shape 1 ". When both sides of the ring cross-section are of half circular shape ("Shape 3 "), the capacitance values are the biggest.

Table 6 Capacitance for different ring cross-section shapes, for $b / a=2.0, c / a=3.0$ and $h / a=0.5$.

| $N=200$ |  | $C / 2 \pi^{2} \varepsilon a$ |  |
| :---: | :---: | :---: | :---: |
| $\Delta$ | Shape 1 | Shape 2 | Shape 3 |
| 0.01 | 0.84128101 | 0.84225911 | 0.84258872 |
| 0.05 | 0.85343273 | 0.85849666 | 0.85998528 |
| 0.10 | 0.86592667 | 0.87603187 | 0.87858828 |
| 0.15 | 0.87696743 | 0.89245478 | 0.89585774 |
| 0.20 | 0.88707376 | 0.90833128 | 0.91240785 |

Table 7 Capacitance for different ring cross-section shapes, for $b / a=2.0, c / a=3.0$ and $h / a=1.0$.

| $N=200$ | $C / 2 \pi^{2} \varepsilon a$ |  |  |
| :---: | :---: | :---: | :---: |
| $\Delta$ | Shape 1 | Shape 2 | Shape 3 |
| 0.01 | 0.79283408 | 0.79354619 | 0.79389070 |
| 0.05 | 0.80313199 | 0.80675523 | 0.80833047 |
| 0.10 | 0.81377517 | 0.82087548 | 0.82361947 |
| 0.15 | 0.82322328 | 0.83393232 | 0.83763635 |
| 0.20 | 0.83191439 | 0.84639990 | 0.85089954 |

Table 8 Capacitance for different ring cross-section shapes, for $b / a=2.0, c / a=3.0$ and $h / a=5.0$.

| $N=200$ |  | $C / 2 \pi^{2} \varepsilon a$ |  |
| :---: | :---: | :---: | :---: |
| $\Delta$ | Shape 1 | Shape 2 | Shape 3 |
| 0.01 | 0.53392498 | 0.53402733 | 0.53436686 |
| 0.05 | 0.53830085 | 0.53876972 | 0.54040538 |
| 0.10 | 0.54276475 | 0.54358484 | 0.54660431 |
| 0.15 | 0.54667231 | 0.54778650 | 0.55209139 |
| 0.20 | 0.55022101 | 0.55158615 | 0.55709905 |

From Table 8 it is evident that an influence of the ring thickness is negligible. When the distance between the ring and the sphere is large, the sphere doesn't "see" the ring thickness. The distance between the EEs placed on the ringsides and their images in the sphere is approximately equal. That is the reason why the ring thickness hasn't influence on the system capacitance.

In Figs. 9-10 the capacitance values for different parameters values are shown. All presented results are obtained when ring cross-section has the "Shape 1". In Fig. 9 the capacitance dependence versus ring thickness, i.e. parameter $\Delta$, when $b / a$ and $c / a$ have constant values, is shown. From Fig. 9b it is evident that when the distance between the ring and the sphere increases and the ring has bigger thickness, the capacitance has constant value. As it is mentioned, in that case, the sphere doesn't "see" the ring thickness, so the capacitance is constant.

The capacitance values for thin ring with the negligible thickness and the ring with the finite thickness have been compared in Fig. 10.


Fig. 9 Capacitance dependence versus parameter $\Delta$ for different values of parameter $h / a$.

(a)

(b)


Fig. 10 Results comparison for different ring thickness and different values of parameters.

From Figs. 10c and 10d, it can be seen that when the distance between ring and sphere increases, the capacitance values stream to equal value.

In Fig. 11, the equipotential curves, obtained using software package [4], are shown.


Fig. 11 Equipotential curves (FEMM 4.0) for $b / a=2.0$,

$$
c / a=3.0 \text { and } h / a=1.0 .
$$

In Tables 9-10, the EEM results have been compared with FEM results. FEM values are obtained using FEMM software package [4]. The ring cross-section shape is rectangular. The number of EEs in EEM is $N=200$. From these tables the good results agreement can be noticed (an error rate is less than $0.5 \%$ ).

Table 9 Capacitance values comparison, for $b / a=2.0$, $c / a=3.0$ and $h / a=0.0$.

|  | $C / 2 \pi^{2} \varepsilon a$ |  |
| :---: | :---: | :---: |
| $\Delta$ | EEM | FEM |
| 0.00 | 0.82815312 | 0.86051667 |
| 0.01 | 0.86140922 | 0.86031416 |
| 0.10 | 0.88775952 | 0.88794107 |
| 0.20 | 0.91024679 | 0.90973715 |

Table 10 Capacitance values comparison, for $b / a=2.0$, $c / a=3.0$ and $h / a=2.0$.

|  | $C / 2 \pi^{2} \varepsilon a$ |  |
| :---: | :---: | :---: |
| $\Delta$ | EEM | FEM |
| 0.00 | 0.68018716 | 0.68509607 |
| 0.01 | 0.68668157 | 0.68521170 |
| 0.10 | 0.70134705 | 0.70027710 |
| 0.20 | 0.71407189 | 0.71278599 |

## 5. CONCLUSION

The obtained results have shown that when the number of equivalent electrodes is small $(N=50)$, the good convergence of the results is achieved. When the distance between the ring and the sphere increases, for any ring cross-section shape, the capacitance has approximately a constant value.

The EEM results are obtained using a program written in FORTRAN 77. The CPU calculation time is connected with the total number of the EEs. When the number of EEs increases, the CPU time increases too. But this calculation time is not so significant. All necessary calculations have been done only for a few seconds. All calculations are carried out on a PC with 256 MB RAM, 1.6 GHz .

The EEM results have been compared with the FEM results. The excellent results agreements have been obtained. The ring is divided in the strips using the expressions (1) and (2) because in the earlier investigation [1] is shown that the obtained error is smaller.

The FEM is based on differential equations solving and domain discretization. On the other side, using the EEM, it is necessary to solve only a system of linear equations. Therefore, the EEM calculation time is shorter than the FEM CPU time. Using the FEM is easy to solve problems having a complex geometry and different interfacial boundaries to a degree of accuracy. Only the closed problems can be solved using the FEM. Using the EEM it is possible to solve open electromagnetic problems.

The procedure presented in this article can be applied in the grounding theory, but that will be the task for further investigation. Also, it should be interesting to investigate an influence of equivalent electrodes arrangement on capacitance values.

The obtained results will be good input data for practical application design of this capacitor. After design of such prototype, it will be possible to compare the calculated values with the measured results.

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Received January 23, 2009, accepted April 5, 2009

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