PREDICTIVE CONTROL ALGORITHM BASED ON STATE SPACE MODELS OF DYNAMIC SYSTEMS - INTERNET APPROACH

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SUMMARY

The paper is focused on Predictive Control Algorithm design for time-invariant state space models of dynamic systems with various dynamics, (mechanical system-helicopter, hydraulical system - three tanks with interaction). The predictive control algorithm based on state space models is verified by simulation schemes in language Matlab/Simulink using architecture of S-functions of the library PredicLib. The results are presented by created Internet applications using the technology Matlab Web Server. Internet applications enable to simplify the usage of designed predictive algorithms for testing their properties at various dynamics of MIMO non-linear systems without the proper installation language Matlab/Simulink on client's PC.

Keywords: *model based predictive control, dynamic system, state space model, equations of predictions, minimization of quadratic criterion, conditions for constrains, virtual laboratory*

1. INTRODUCTION

The method of predictive control is a powerful model-based approach for control of processes from industry, which enables user to handle different control requirements (limits, constraints).

Model Predictive Control (MPC) is the method of control in which the goal is to compute such a future control sequence, that the future system output is driven as close as possible the reference trajectory. This is accomplished by minimizing quadratic multistage cost function defined over the prediction horizon [4], [5].

The key features of MPC are:

- an explicit using of a model of dynamic system to predict the system output at the future time (horizon of prediction),
- the calculation of a control sequence by minimizing of the cost function,
- the receding strategy - calculation of the actual manipulated variable in each control step.

The various MPC algorithms differ among themselves in the model used to represent the process and the cost function to be minimized. By Generalized Predictive Control (GPC) algorithm plant model is often in form of the rational transfer function. In this case the solution of Diophantine equations is required to compute prediction of the system output [1], [2].

The predictive control algorithm presented in this paper is based on the mathematical-physical models of the controlled dynamic systems in the state space (mechanical system - helicopter, hydraulic system - three tanks with interaction). For using a state space formulation of the predictive control algorithm the states of the system are necessary to be known. When, the states of the system are not available (are not measured), then the states of the system are estimated by the linear state space observer based on Kalman estimator [1].

The paper is focused on the predictive control algorithm design for time-invariant state space models with various dynamics. The predictive control algorithm is verified by simulation schemes in language Matlab/Simulink using architecture of S-functions of the library PredicLib [7]. The results are presented by created an internet applications using technology Matlab Web Server (MWS) [7], [10].

2. PREDICTIVE CONTROL BASED ON STATE SPACE MODEL OF THE SYSTEM

The Model Predictive Control is a multistage approach, combining feedforward and feedback control design.

The feedforward control design is represented by the predictions based on the mathematical model of the dynamic system. This part is dominant component of the control actions. The feedback control design from measured outputs serves for compensation of some bounded model inaccuracies and low external disturbances. The design consists from local minimization of quadratic criterion (1) in which the predictions from the equations of the predictions are involved. The minimization is repeated in each time step [4].

The cost function for predictive control calculating for *k*-th step can be represented as

$$J_{MPC}(k) = \sum_{i=1}^{n}\sum_{j=0}^{N_p}\mu_i\left(\hat{y}_i(k+j\,|\,k) - r_i(k+j\,|\,k)\right)^2 + \sum_{l=1}^{m}\sum_{j=0}^{N_p}\lambda_l u_l^2(k+j\,|\,k) \qquad (1)$$

where N_p is predictive horizon, μ_i, λ_l are the positive weight coefficients for the output and the input, $\hat{y}_i(k+j\,|\,k)$ is predicted system output value, $r_i(k+j\,|\,k)$ is required system output (reference) value for *i*-th output of the system.

The first term in the performance criterion refers to the square variation of the predicted system output from the desired reference trajectory, while the second term is added in order to limit the controller output, greater λ yields less active controller output.

The predictive control algorithm design is based on the linearized equations of discrete state space model of the dynamic system, which will be used to compute of the predictor.

Let us consider linear discrete state space model of the dynamic system

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)+\boldsymbol{D}\boldsymbol{u}(k) \end{aligned} \qquad (2)$$

where $\boldsymbol{x}(k)$ denotes the state vector (*dimension n*), $\boldsymbol{y}(k)$ denotes the system outputs or measurements to be controlled (*dimension n*) and $\boldsymbol{u}(k)$ (*dimension m*) denotes the system inputs (or the controller outputs), $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are the matrices defining the state space model. In generally for the real systems the matrix $\boldsymbol{D}=0$.

The model maps interval of one sampling period. Principle of the equations of predictions is expression of future values of the outputs $\boldsymbol{y}(k)$ from the current measured state $\boldsymbol{x}(k)$ as follows

$$\begin{aligned} \hat{\boldsymbol{y}}(k) &= \boldsymbol{C}\boldsymbol{x}(k) \\ \hat{\boldsymbol{x}}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\boldsymbol{u}(k) \\ \hat{\boldsymbol{y}}(k+1) &= \boldsymbol{C}\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{C}\boldsymbol{B}\boldsymbol{u}(k) \\ &\vdots \\ \hat{\boldsymbol{x}}(k+N_p) &= \boldsymbol{A}^{N_p}\boldsymbol{x}(k)+\boldsymbol{A}^{N_p-1}\boldsymbol{B}\boldsymbol{u}(k)+ \\ &\quad +\cdots+\boldsymbol{B}\boldsymbol{u}(k+N_p-1) \\ \hat{\boldsymbol{y}}(k+N_p) &= \boldsymbol{C}\boldsymbol{A}^{N_p}x(k)+\boldsymbol{C}\boldsymbol{A}^{N_p-1}\boldsymbol{B}\boldsymbol{u}(k)+ \\ &\quad +\cdots+\boldsymbol{C}\boldsymbol{B}\boldsymbol{u}(k+N_p-1) \end{aligned} \qquad (3)$$

Equations (3) can be usefully written in the matrix notation

$$\hat{\boldsymbol{y}} = \boldsymbol{G}\boldsymbol{u}+\boldsymbol{S}\boldsymbol{x}(k) \qquad (4)$$

where

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{\boldsymbol{y}}(k+1) \\ \hat{\boldsymbol{y}}(k+2) \\ \vdots \\ \hat{\boldsymbol{y}}(k+N_p) \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}(k) \\ \boldsymbol{u}(k+1) \\ \vdots \\ \boldsymbol{u}(k+N_p-1) \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{CB} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{CAB} & \boldsymbol{CB} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \boldsymbol{0} \\ \boldsymbol{CA}^{N_p-1}\boldsymbol{B} & \boldsymbol{CA}^{N_p-2}\boldsymbol{B} & \cdots & \boldsymbol{CB} \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \vdots \\ \boldsymbol{CA}^{N_p} \end{bmatrix}$$

This is the base of the predictive control – new unknown values are obtained from actual values.

2.1. Computing of an optimal control

For computing of an optimal control the cost function (1) can be written in matrix notation:

$$J_{MPC} = \left(\hat{\boldsymbol{y}}-\boldsymbol{r}\right)^T \boldsymbol{M}\left(\hat{\boldsymbol{y}}-\boldsymbol{r}\right)+\boldsymbol{u}^T\boldsymbol{L}\boldsymbol{u} \qquad (5)$$

where

$$\boldsymbol{r} = \begin{bmatrix} \boldsymbol{r}(k+1) \\ \boldsymbol{r}(k+2) \\ \vdots \\ \boldsymbol{r}(k+N_p) \end{bmatrix}, \quad \boldsymbol{M} = \begin{bmatrix} \boldsymbol{\mu}(k+1) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\mu}(k+2) & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\mu}(k+N_p) \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{\lambda}(k+1) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\lambda}(k+2) & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\lambda}(k+N_p) \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{\mu}(k+j) &= diag\left(\begin{bmatrix}\mu_1 & \mu_2 & \cdots & \mu_n\end{bmatrix}\right) \\ \boldsymbol{\lambda}(k+j) &= diag\left(\begin{bmatrix}\lambda_1 & \lambda_2 & \cdots & \lambda_m\end{bmatrix}\right) \end{aligned}$$

for $j = 0, 1, \ldots, N_p$.

The predictor written by the matrix notation (4) can be substituted in the cost function (5) for an optimal control computing. We obtain:

$$\begin{aligned} J_{MPC} &= \left(\boldsymbol{S}\boldsymbol{x}(k)+\boldsymbol{G}\boldsymbol{u}-\boldsymbol{r}\right)^T \boldsymbol{M}\left(\boldsymbol{S}\boldsymbol{x}(k)+\boldsymbol{G}\boldsymbol{u}-\boldsymbol{r}\right) \\ &\quad +\boldsymbol{u}^T\boldsymbol{L}\boldsymbol{u} \end{aligned} \qquad (6)$$

After simplifying the equation (6) the cost function can be written in the following form:

$$J_{MPC} = \frac{1}{2}\boldsymbol{u}^T\boldsymbol{H}\boldsymbol{u}+\boldsymbol{b}^T\boldsymbol{u}+\boldsymbol{f}_0 \qquad (7)$$

where

$$\begin{aligned} \boldsymbol{H} &= 2\left(\boldsymbol{G}^T\boldsymbol{M}\boldsymbol{G}+\boldsymbol{L}\right) \\ \boldsymbol{b}^T &= 2\left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right)^T\boldsymbol{G}\boldsymbol{M} \\ \boldsymbol{f}_0 &= \left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right)^T\left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right) \end{aligned}$$

The minimum of the cost function J_{MPC} (7) can be found by making gradient of J_{MPC} equal to zero, which leads to:

$$\boldsymbol{u} = -\boldsymbol{H}^{-1}\boldsymbol{b} = -\left(\boldsymbol{G}^T\boldsymbol{M}\boldsymbol{G} + \boldsymbol{L}\right)^{-1}\boldsymbol{G}^T\boldsymbol{M}\left(\boldsymbol{S}\boldsymbol{x}(k) - \boldsymbol{r}\right) \quad (8)$$

The result of the equation (8) is the trajectory consisting of optimal control inputs, where the first *m* of them are applied on the system and is given by:

$$\boldsymbol{u}(k) = \boldsymbol{K}\left(\boldsymbol{r} - \boldsymbol{S}\boldsymbol{x}(k)\right) \quad (9)$$

where $\boldsymbol{K}$ is the first *m*-rows of the matrix $\left(\boldsymbol{G}^T\boldsymbol{M}\boldsymbol{G} + \boldsymbol{L}\right)^{-1}\boldsymbol{G}^T\boldsymbol{M}$.

For the dynamic system with the constrains for controller output value or the system output value, vector of the optimal control $\boldsymbol{u}$ is calculating by Optimization Toolbox function **quadprog** of language Matlab:

$\boldsymbol{u} = quadprog(\boldsymbol{H}, \boldsymbol{b}^T, \boldsymbol{L}_{CON}, \boldsymbol{v}, \boldsymbol{U}_{\min}, \boldsymbol{U}_{\max})$

where

$\boldsymbol{U}_{\min} \le \boldsymbol{u} \le \boldsymbol{U}_{\max}$ and $\boldsymbol{L}_{CON}\boldsymbol{u} \le \boldsymbol{v}$.

The vectors $\boldsymbol{U}_{\min}$ and $\boldsymbol{U}_{\max}$ are the column vectors those elements are minimal and maximal values of $\boldsymbol{u}(k)$. With using the matrix $\boldsymbol{L}_{CON}$ and the vector $\boldsymbol{v}$ can be defined a system of inequalities that insure so constrain conditions will be satisfied [2], [3].

2.2. Implementation of predictive control algorithm based on the state space model to Matlab

The algorithm of predictive control for MIMO dynamic system was designed with using S-functions in programmable environment Matlab/Simulink.

The algorithm for the calculating of the control input value in *k*-th step:

1. reading of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ of the linearized discrete state space model (2) of dynamic system, the reference vector trajectory $\boldsymbol{r}$ and the estimated state vector $\boldsymbol{x}_e(k)$,
2. creating of the weight matrices $\boldsymbol{M}$ and $\boldsymbol{L}$,
3. calculate the matrix of the free response $\boldsymbol{S}$ and the matrix of the force response $\boldsymbol{G}$,
4. if required constrains for the values of the control input $\boldsymbol{u}(k)$, then continue by the step 8,
5. calculating of the feedback gain of the control matrix $\boldsymbol{K}$,
6. calculating of the controller output by $\boldsymbol{u}(k) = \boldsymbol{K}(\boldsymbol{r} - \boldsymbol{S}\boldsymbol{x}_e(k))$,
7. continue by the step 12,
8. creating of the vectors $\boldsymbol{U}_{\min}$ and $\boldsymbol{U}_{\max}$,
9. calculating Hessian matrix $\boldsymbol{H}$ and the vector $\boldsymbol{b}^T$,
10. $\boldsymbol{u} = quadprog(\boldsymbol{H}, \boldsymbol{b}^T, [\,], [\,], \boldsymbol{U}_{\min}, \boldsymbol{U}_{\max})$,
11. $\boldsymbol{u}(k)$ consists from the first *m*-rows of the control vector $\boldsymbol{u}$,
12. the application of the control signal $\boldsymbol{u}(k)$ on the system input.

3. SIMULATION VERIFICATION OF MPC ALGORITHM FOR DYNAMIC SYSTEMS

Designed MPC algorithm based on the state space model was applicated on simulation models as mechanical system-helicopter and hydraulic system-three tanks with interaction.

For verification of the predictive algorithm by designed control structure on Fig. 1. in language Matlab/Simulink was created model of helicopter as S-function in the state space for an equilibrium point where as non-linear mathematical-physical model of helicopter was used model, which is derived in [6]. Functional block of MPC controller is included in the library PredicLib [3].

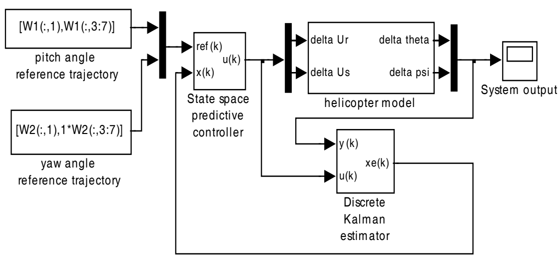

Fig. 1 The control simulation scheme of the predictive control based on state space model of the system

The mechanical system-helicopter has two degrees of freedom, the rotation of the helicopter body with respect to the horizontal axis (pitch angle θ) and the rotation around the vertical axis (yaw angle ψ), which are measured by two sensors. The helicopter can move from $\langle$-170°,170°$\rangle$ in yaw, and from $\langle$-45°,45°$\rangle$ in pitch.

The inputs to the model are the voltages U_R and U_S affecting the main and the rear rotor. Both the inputs are constrained between $-10V$ and $10V$.

For linearization of the nonlinear differential equations of the helicopter at an equilibrium point by $\theta = 0$ and $\psi = 0$ the following measurements were done at input voltages $U_{RE} = 1.7V$, $U_{SE} = 3V$ of the steady state of the lab helicopter model [6]. At the voltages U_{RE} and U_{SE} in a equilibrium point the system parameters are:

$$\begin{aligned} &\omega_{RE} = 57 \ [rad/s] && \theta_E = 0 \ [rad] \\ &\omega_{\theta E} = 0 \ [rad/s] && \omega_{SE} = 100 \ [rad/s] \qquad (10) \\ &\psi_E = 0 \ [rad] && \omega_{\psi E} = 0 \ [rad/s] \end{aligned}$$

where

$U_R = U_{RE} + \Delta U_R \qquad U_S = U_{SE} + \Delta U_S$

$\omega_R = \omega_{RE} + \Delta\omega_R \qquad \omega_S = \omega_{SE} + \Delta\omega_S$

We can find linear-time-invariant (LTI) model by linearizing the non-linear differential equations from [6] with the values of an equilibrium point and system parameters (10) by Jacobian matrices

$$\boldsymbol{A}_C = \left[\frac{\partial \boldsymbol{f}_i}{\partial x_j}\right]_{\boldsymbol{x}=\boldsymbol{x}_E, \boldsymbol{u}=\boldsymbol{u}_E}, \quad \boldsymbol{B}_C = \left[\frac{\partial \boldsymbol{f}_i}{\partial u_j}\right]_{\boldsymbol{x}=\boldsymbol{x}_E, \boldsymbol{u}=\boldsymbol{u}_E} \qquad (11)$$

The LTI continuous model of the helicopter can now be expressed as

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_C \boldsymbol{x}(t) + \boldsymbol{B}_C \boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}_C \boldsymbol{x}(t) \end{aligned} \qquad (12)$$

where $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$ are the state, the input and the output vectors.
The matrices $\boldsymbol{A}_C$, $\boldsymbol{B}_C$ and $\boldsymbol{C}_C$ of the state space helicopter model (12) have structure

$$\boldsymbol{A}_C = \begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ A_{61} & 0 & 0 & A_{64} & 0 & A_{66} \end{bmatrix}, \quad \boldsymbol{B}_C = \begin{bmatrix} B_{11} & 0 \\ 0 & 0 \\ 0 & B_{32} \\ 0 & B_{42} \\ 0 & 0 \\ B_{61} & 0 \end{bmatrix}$$

$$\boldsymbol{C}_C = \begin{bmatrix} 0 & C_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and their elements are defined in [6] and also in *CyberVirtLab* [8].

Further the state vector $\boldsymbol{x}(t)$ and the input vector $\boldsymbol{u}(t)$ are

$$\boldsymbol{x}(t) = \left[\Delta\omega_R(t) \;\; \theta(t) \;\; \omega_\theta(t) \;\; \Delta\omega_S(t) \;\; \psi(t) \;\; \omega_\psi(t)\right]^T$$

$$\boldsymbol{u}(t) = \left[u_1(t) \;\; u_2(t)\right]^T = \left[\Delta U_R(t) \;\; \Delta U_S(t)\right]^T$$

where

$\Delta\omega_R(t)$ - deviation from the main rotors angular velocity in the equilibrium point

$\theta(t)$ - pitch angle,

$\omega_\theta(t)$ - pitch velocity,

$\Delta\omega_S(t)$ - deviation from the rear rotors angular velocity in equilibrium point,

$\psi(t)$ - yaw angle,

$\omega_\psi(t)$ - yaw velocity,

$\Delta U_R(t)$ - deviation from the voltage equilibrium point for the main rotor,

$\Delta U_S(t)$ - deviation from the voltage equilibrium point for the rear rotor.

The sensors are measuring the two states $\theta(t)$ and $\psi(t)$. Taking derivative of $\theta(t)$ and $\psi(t)$, we also have the values for $\omega_\theta(t)$ and $\omega_\psi(t)$. The physical value of the measured output will be:

$y_1(t)$ - voltage from pitch sensor measuring θ $[V]$,

$y_2(t)$ - pitch velocity ω_θ $[\mathrm{rad/sec}]$,

$y_3(t)$ - voltage from yaw sensor measuring ψ $[V]$,

$y_4(t)$ - yaw velocity ω_ψ $[\mathrm{rad/sec}]$.

The discretization of the linearized model of the helicopter (12) can be done in the language Matlab using function **c2d** at the chosen sampling time T_S.

Parameters of the simulation for MPC algorithm as:

- the equilibrium point of the helicopter model,
- the sampling period T_S, ($T_S = 0.01s$),
- the prediction horizon N_P for the output of the system, (we consider $N_P = 10$ when the constrains on the output of the predictive controller are defined),
- the constrains for controller output value $\boldsymbol{U}_{R\min}$, $\boldsymbol{U}_{R\max}$, $\boldsymbol{U}_{S\min}$, $\boldsymbol{U}_{S\max}$,
- the weight coefficients of matrices $\boldsymbol{M}$ and $\boldsymbol{L}$ of the cost function J_{MPC} (1)

can be defined by an Internet form [8], which is included in *CyberVirtLab* [7].

The results of the tracking of the reference trajectories $r_1(t), r_3(t)$ by the system's outputs $y_1(t) = \theta(t)$, $y_3(t) = \psi(t)$ using MPC algorithm are on Fig. 2, whereas on the output of the system have effect disturbance in form of the white noise that simulates of the measurement error.

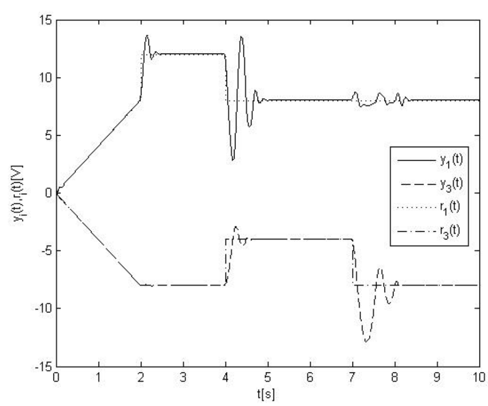

Fig. 2 The tracking of the reference trajectories $r_1(t), r_3(t)$ by real outputs of the system $y_1(t)$, $y_3(t)$

The output of MPC controller – an optimal control signals $u_1(t), u_2(t)$ are on Fig. 3.

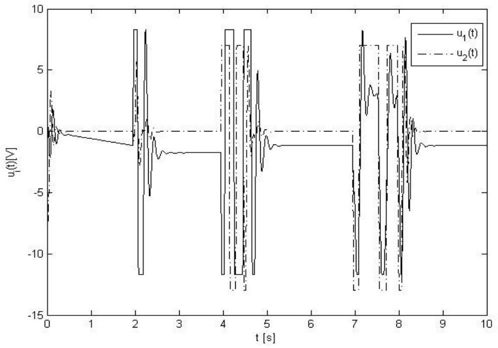

Fig. 3 The MPC controller outputs – control signals $u_1(t), u_2(t)$

The results of MPC strategy with the constrains on the control inputs, which are illustrated on Fig. 2 and Fig. 3 (with added white noise to the output of the system) shows on the tracking of the reference trajectories when was used discrete linear Kalman estimator [3].

In a similar way is MPC algorithm applicated on hydraulic system of the three tanks with an interaction whereas for computing of the predictor (4) was used linearized model of MIMO system in the state space. Unlike from the mechanical system – helicopter, which is unstable with quick dynamics, the hydraulic system is stable with slow dynamic.

The model of the hydraulic system consists from three cylindrical tanks with the same cylindrical tank bottom area F, which are interconnected at the bottom by pipes. The outflows from the tanks are controlled by relative open of value V_i.

The outflow from the first tank is the inflow to the second one, the picture of the system is showed on Internet - *CyberVirtLab* [9].
For changing levels $H_1(t)$, $H_2(t)$ and $H_3(t)$ holds:

$$\begin{aligned} F_1\rho\frac{dH_1(t)}{dt} &= \rho M_1(t) - \rho M_2(t) \\ F_2\rho\frac{dH_2(t)}{dt} &= \rho M_2(t) - \rho M_3(t) \\ F_3\rho\frac{dH_3(t)}{dt} &= \rho M_3(t) - \rho M_4(t) \end{aligned} \tag{13}$$

From the physics (the part of the hydraulics) is known the equation for calculation the mass flow between tanks (ρ is the density of the liquid):

$$M_{i+1}(t) = \frac{f_i\rho\sqrt{2g}.\sqrt{H_i(t)-H_{i-1}(t)}}{F_i}k_{vi}\,f\left(u_i(t)\right) \tag{14}$$
$i = 1,...,3$

where

k_{vi} is constructing constant of the outlet of the cylindrical tank,
$f\left(u_i(t)\right)$ is characteristic function of the valve V_i,
$M_1(t)$ $[kg/s]$ is the mass inflow to cylindrical tank,
$M_i(t)$ $[kg/s]$ is *i*-th mass flow between tanks through the valve V_i,
$H_i(t)$ $[m]$ denotes the water level in the *i*-th cylindrical tank,
F $[m^2]$ is the cylindrical tank bottom area,
f_i $[m^2]$ is the area outflow of the *i*-th cylindrical tank,
$u_i(t)$ $[mm]$ is the rises of the output outlet of the cylindrical tank, for $i = 1,...,3$.

After inducting the equation (14) to the equation (13) whereas we consider, that function $f\left(u_i(t)\right)$ will be linear, we obtain the non-linear differential equations (15) describing dynamics of the changing levels $H_i(t)$ in the tanks:

$$\begin{aligned} \frac{dH_1(t)}{dt} &= \frac{M_1(t)}{F_1} - \frac{f_1\sqrt{2g}\sqrt{H_1(t)-H_2(t)}}{F_1}k_{v1}u_1(t) \\ \frac{dH_2(t)}{dt} &= \frac{f_1\sqrt{2g}\sqrt{H_1(t)-H_2(t)}}{F_2}k_{v1}u_1(t) - \\ &\quad - \frac{f_2\sqrt{2g}\sqrt{H_2(t)-H_3(t)}}{F_2}k_{v2}u_2(t) \\ \frac{dH(t)_3}{dt} &= \frac{f_2\sqrt{2g}\sqrt{H_2(t)-H_3(t)}}{F_3}k_{v2}u_2(t) - \\ &\quad - \frac{f_3\sqrt{2g}\sqrt{H_3(t)}}{F_3}k_{v3}u_3(t) \end{aligned} \tag{15}$$

For the set-point state of the equations (15), that is when $dH_i(t)/dt = 0$, for $i = 1,...,3$ and by expansion to Taylor series for the set-point $SP = [H_{10}\ H_{20}\ H_{30}\ u_{10}\ u_{20}\ u_{30}]$ we obtain linearized state space model (12),
where the structure of the matrices $\boldsymbol{A}_C$, $\boldsymbol{B}_C$, $\boldsymbol{C}_C$ is

$$\boldsymbol{A}_C = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix}, \boldsymbol{B}_C = \begin{bmatrix} B_{11} & 0 & 0 \\ B_{21} & B_{22} & 0 \\ 0 & B_{32} & B_{33} \end{bmatrix}$$

$$\boldsymbol{C}_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

the state vector $\boldsymbol{x}(t) = \left[\Delta H_1(t), \Delta H_2(t), \Delta H_3(t)\right]^T$ and the control vector $\boldsymbol{u}(t) = \left[\Delta u_1(t) \quad \Delta u_2(t) \quad \Delta u_3(t)\right]^T$.

The MPC algorithm was applicated on the hydraulic system, which is created as S-function, and verified by modified control structure on Fig. 1 in the language Matlab/Simulink.

The parameters of the simulation for MPC algorithm as:

- the equilibrium point of the hydraulic system,
- the sampling period T_S , ($T_S = 0.2s$),
- the prediction horizon N_P for the output of the system, ($N_P = 5$),
- the weight coefficients of the matrices $\boldsymbol{M}$ and $\boldsymbol{L}$, of the cost function J_{MPC} (1)

can be defined by an Internet form [9], which is included in *CyberVirtLab* [7].

The presentation of the results of MPC strategy is illustrated on Fig. 4 and Fig. 5.

The plots on Fig. 4 compare the reference output of the system $r_i(t)$ and the actual output of the closed loop system $\Delta H_i(t)$.

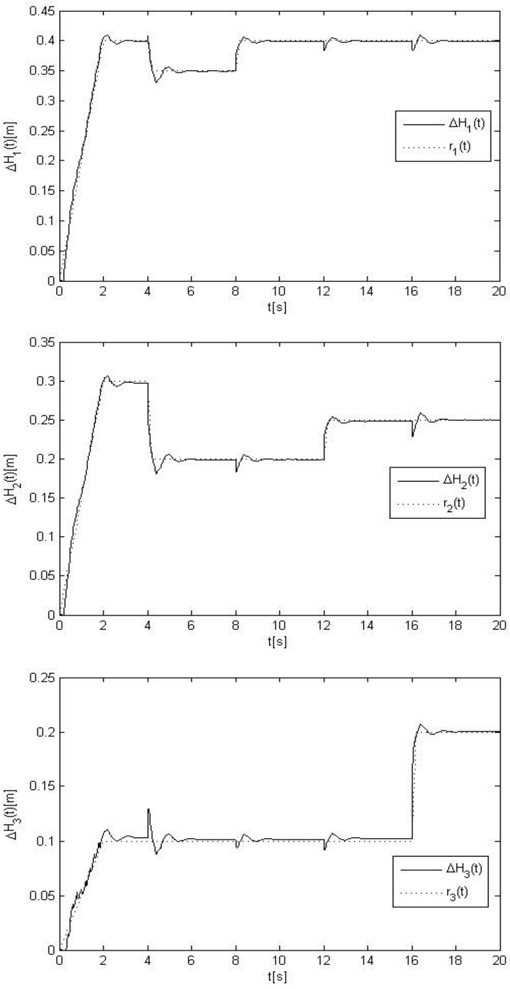

Fig. 4 The graphic representation of the reference outputs $r_i(t)$ and the real outputs of the hydraulic system $\Delta H_i(t)$

The plot in Fig.5 shows the results of MPC algorithm – an optimal control signals $\Delta u_1(t)$, $\Delta u_2(t)$, $\Delta u_3(t)$.

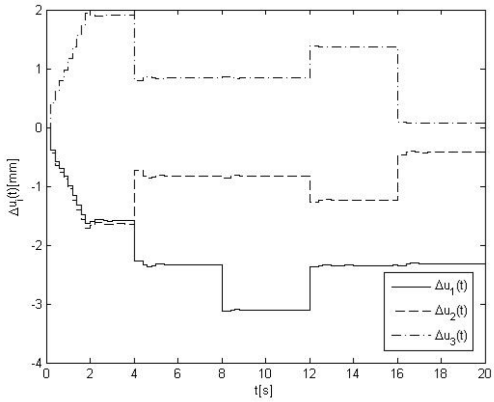

Fig. 5 The MPC controller outputs – control signals $\Delta u_1(t)$, $\Delta u_2(t)$, $\Delta u_3(t)$

4. CONCLUSION

This paper presents the approach of an internet verification MPC algorithm based on the state space model of the dynamical system (mechanical system - helicopter and hydraulic system) by the simulation schemes in language Matlab/Simulink using the technology Matlab Web Server.

These created an internet applications can be used on the exercises of courses as Theory Optimal and Adaptive Systems, Control and Artificial Intelligence and Control and Process Visualization, which are lectured on the Department of Cybernetics and Artificial Intelligence of Faculty of Electrical Engineering and Informatics, Technical University of Košice.

An internet applications enable to simplify the usage of designed MPC algorithms for the testing their properties at the various dynamics of MIMO non-linear systems without the proper installation simulation language Matlab/Simulink on the client's PC.

The authors of this paper have attempted to an introduce the technology Matlab Web Server as suitable means of motivating students within the courses such as Control Theory and Visualization by the simulation verification of an optimal algorithm design in Matlab/Simulink environment.

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