COMPUTATION OF HARMONIC FLOWS IN THREE-PHASE SYSTEMS

*Martin KANÁLIK, **Michal KOLCUN
*Department of Electric Power Engineering, Faculty of Electrical Engineering and Informatics,
Technical University of Košice, Masiarska 74, 042 01 Košice, tel. 055/602 3560, E-mail: martin.kanalik@tuke.sk
** Department of Electric Power Engineering, Faculty of Electrical Engineering and Informatics,
Technical University of Košice, Masiarska 74, 042 01 Košice, tel. 055/602 3550, E-mail: michal.kolcun@tuke.sk

SUMMARY

This paper deals with computation of harmonic current flows and voltages in three phase systems using by a nodal method. The nodal method is useful to determinate unknown branch currents and nodal voltages by known source voltages or currents and nodal admittance matrix of the network. To obtain the correct solution it is necessary to correctly determinate the nodal admittance matrix, which contains admittances of three-phase elements. This article is also describing the ways how to model some three-phase elements for the purpose of nodal method analysis.

Keywords: *harmonics flows, nodal method for three-phase systems, three-phase elements modelling.*

1. INTRODUCTION

Although the title specially refers to harmonic flows, the analysis described in this article is equally applicable to other frequencies in the region of interest such as inter-harmonics and subharmonics.

The simplest harmonic flow involves a single harmonic source and single-phase network analysis. This model is commonly used to derive the system harmonic impedances at the point of common coupling in filter design, In general; however, the network will be unbalanced and may contain several harmonic sources. Therefore, the derivation of the harmonic voltages and currents requires multi-source three-phase harmonic analysis [1].

2. NODAL ANALYSIS IN THREE-PHASE SYSTEMS

The distribution of voltage and current harmonics throughout a linear power network containing one or more harmonic current sources is normally carried out using nodal analysis [2]. The asymmetry inherent in transmission systems cannot be studied with any simplification by using the symmetrical component frame of reference; therefore phase components are used [1].

The nodal admittance matrix of the network at frequency f is of the form

$$\left[Y_f\right]=\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2k} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots & & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{ik} & \cdots & Y_{in} \\ \vdots & \vdots & & \vdots & \ddots & \vdots & & \vdots \\ Y_{k1} & Y_{k2} & \cdots & Y_{ki} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \vdots & & \vdots & & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix} \qquad (1)$$

where Y_{ki} is the mutual admittance between busbars k and i at frequency f, and Y_{ii} is the self-admittance of busbar i at frequency f.

The three-phase nature of the power system always results in some load of transition line asymmetry, as well as circuit coupling. These effects give rise to unbalanced self- and mutual admittances of the network elements.

For the three-phase system, the elements of the admittance matrix are themselves 3x3 matrices consisting of self- and transfer admittances between phases, i.e.

$$Y_{ii}=\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix} \qquad (2)$$

The nodal method is based on the solution of equation (3):

$$\left[I_{bus}\right]=\left[Y_{bus}\right]\cdot\left[U_{bus}\right] \qquad (3)$$

Consider the three-phase network shown on figure. 1, which is a simple example of network, where the line impedance is considered only as a parallel connection of series RL components, which values are equal for Z1, Z2, Z3 and Z4.

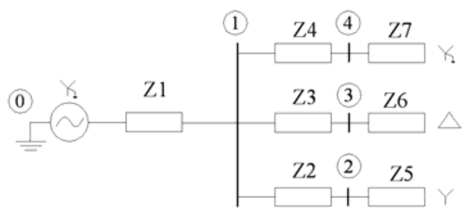

Fig. 1 Example of three-phase network – single-phase equivalent

Impedances Z5, Z6 and Z7 represents three-phase loads with different connection, i.e. Z5 is wye-connected load, Z6 is delta-connected load and Z7 is grounded wye-connected load.

One single way how to model such network is to model it using by a single-phase method as describes figure 2. One can see the difference in number of nodes between figure 1 and figure 2. The reason is that there is one node for each phase of the network in figure 2 instead of one node for all three phases as shows figure 1. Because there are three phases for each node in figure 1, the real number of nodes is:

$$N_{3f} = 3\times N_{1f} + N_y - 2 \tag{4}$$

where N_{3f} is number of nodes of three-phase equivalent, N_{1f} is number of nodes of single-phase equivalent model and N_y is number wye-connected elements.

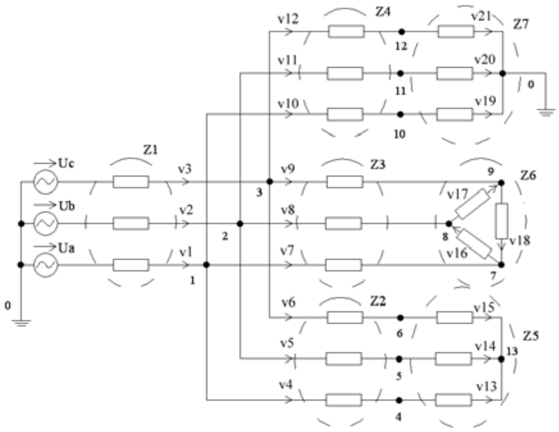

Fig. 2 Example of three-phase network – three-phase equivalent

In a first step for calculation of branch currents and busbars voltages is to correctly determinate the nodal admittance matrix $[Y]$, which can be obtained from equation (5):

$$[Y]=[A]^T[Y_d][A] \tag{5}$$

where $[A]$ is incidence matrix, for this case:

$$[A]=\begin{array}{r} \\ v1\\ v2\\ v3\\ v4\\ v5\\ v6\\ v7\\ v8\\ v9\\ v10\\ v11\\ v12\\ v13\\ v14\\ v15\\ v16\\ v17\\ v18\\ v19\\ v20\\ v21 \end{array}\begin{array}{c} \begin{array}{c|ccc|ccc|ccc|ccc|c} 0&1&2&3&4&5&6&7&8&9&10&11&12&13 \end{array}\\ \left[\begin{array}{c|ccc|ccc|ccc|ccc|c}
-1&1&0&0&&&&&&&&&&\\
-1&0&1&0&&&&&&&&&&\\
-1&0&0&1&&&&&&&&&&\\ \hline
&-1&0&0&1&0&0&&&&&&&\\
&0&-1&0&0&1&0&&&&&&&\\
&0&0&-1&0&0&1&&&&&&&\\ \hline
&-1&0&0&&&&1&0&0&&&&\\
&0&-1&0&&&&0&1&0&&&&\\
&0&0&-1&&&&0&0&1&&&&\\ \hline
&-1&0&0&&&&&&&1&0&0&\\
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&&&&&&&1&0&-1&&&&\\ \hline
1&&&&&&&&&&-1&0&0&\\
1&&&&&&&&&&0&-1&0&\\
1&&&&&&&&&&0&0&-1&
\end{array}\right] \end{array} \tag{6}$$

and $[Y_d]$ is diagonal matrix of three-phase admittances and is determined by equation (7):

$$[Y_d]=\begin{bmatrix} Y_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & Y_2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Y_4 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & Y_5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & Y_6 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & Y_7 \end{bmatrix} \tag{7}$$

where Y_i represents admittances of three-phase elements and are determined by diagonal matrices of relevant branch admittances.

This example can be used for describing of harmonic flows, which depends on three-phase elements connections.

Consider a three-phase network shown on the figure 1 and figure 2, which admittances are:

$$Y_i = \frac{1}{0.05\sqrt{h} + j\omega h 10^{-3}}\begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix} \qquad \text{for } i = 1,2,3,4$$

$$Y_i = \frac{1}{10\sqrt{h} + j\omega h 10^{-3}}\begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix} \qquad \text{for } i = 5,6,7$$

where h is the harmonic order and $\omega = 2\pi f$.

Table 1 and table 2 shows results of branch currents and nodal voltages calculation for fundamental frequency with $U_a = 230e^{j0°}$ V, $U_b = 230e^{-j120°}$ V, $U_c = 230e^{j120°}$ V, 3rd harmonic with $U_a = 10e^{j0°}$ V, $U_b = 10e^{j0°}$ V, $U_c = 10e^{j0°}$ V, and 5th harmonic with $U_a = 15e^{j0°}$ V, $U_b = 15e^{j120°}$ V, $U_c = 15e^{-j120°}$ V.

Branch	1. harmonic		3. harmonic		5. harmonic	
	rms	angle	rms	angle	rms	angle
1	107.65	-13.93	0.56	-9.18	2.82	-28.91
2	107.65	-133.93	0.56	-9.18	2.82	91.09
3	107.65	106.07	0.56	-9.18	2.82	31.09
4	21.74	-11.86	0	5.25	0.56	-24.46
5	21.74	-131.86	0	95.25	0.56	95.53
6	21.74	108.14	0	0	0.56	35.53
7	64.21	-15.34	0	91.67	1.67	-31.98
8	64.21	-135.34	0	98.83	1.67	88.02
9	64.21	104.66	0	63.24	1.67	28.02
10	21.74	-11.86	0.56	-9.18	0.56	-24.46
11	21.74	-131.86	0.56	-9.18	0.56	95.53
12	21.74	108.14	0.56	-9.18	0.56	35.53
13	21.74	-11.86	0	-12.58	0.56	-24.46
14	21.74	-131.86	0	-0.73	0.56	95.53
15	21.74	108.14	0	-0.46	0.56	35.53
16	37.07	14.66	0	-23.67	0.96	-61.98
17	37.07	-105.34	0	33.75	0.96	58.02
18	37.07	134.66	0	-172.49	0.96	-1.98
19	21.74	-11.86	0.56	-9.18	0.56	-24.45
20	21.74	-131.86	0.56	-9.18	0.56	95.53
21	21.74	108.14	0.56	-9.18	0.56	35.53

Tab. 1 Current flows for different harmonics in A

node	1. harmonic		3. harmonic		5. harmonic	
	rms	angle	rms	angle	rms	angle
1	218.91	171.72	9.88	3.00	13.12	16.51
2	218.91	51.72	9.88	3.00	13.12	256.51
3	218.91	-68.28	9.88	3.00	13.12	136.51
4	217.51	169.94	9.88	3.00	12.96	20.45
5	217.51	49.94	9.88	3.00	12.96	260.45
6	217.51	70.06	9.88	3.00	12.96	140.45
7	214.15	166.46	9.88	3.00	12.49	27.96
8	214.15	46.46	9.88	3.00	12.49	267.96
9	214.15	-73.54	9.88	3.00	12.49	147.96
10	217.51	169.94	9.79	6.07	12.96	20.45
11	217.51	49.94	9.79	6.07	12.96	260.45
12	217.51	-70.06	9.79	6.07	12.96	140.45
13	0.00	153.43	9.88	3.00	0.00	36.87

Tab. 2 Nodal voltages for different harmonics in V

The results were obtained solving by equation (8), which represents nodal method in case of known branch admittances and source voltages.

$$\left[U_{branch}\right] = -\left[Y\right]^{-1}\left[A\right]^{T}\left[Y_d\right]\left[U_{source}\right] \tag{8}$$

where U_{source} is vector of source voltages.

3. THREE-PHASE ELEMENTS MODELLING

It is necessary to tell that example in previous chapter was very simple. The three-phase components of the network have n general more complicated impedance expression. This chapter deals with some examples of more-accurately modeling of three-phase components.

3.1. Generator modelling

In presence of harmonics, the impedance of a generator neglecting skin effect will be

$$Z = \begin{cases} Z_0(h) = R_a + jhX_0, & h = 3n = 3,6,9,12,15,\ldots \\ Z_1(h) = R_a + jhX''_d, & h = 3n + 1 = 1,4,7,10,\ldots \\ Z_2(h) = R_a + jhX_2, & h = 3n - 1 = 2,5,8,11,\ldots \end{cases}$$

where

R_a	is the armature resistance, Ω/phase
X''_d	is the subtransient reactance, Ω/phase
X_2	is the negative-sequence reactance, Ω/phase
X_0	is the zero-sequence reactance, Ω/phase
h	is the harmonic order

Taking skin effect into consideration, the armature reactance becomes [3]

$$R_a\left(h\right) = \sqrt{h}R_a \tag{9}$$

3.2. Shunt capacitor banks modelling

A shunt capacitor bank can be represented by the matrix

$$C_{abc} = \begin{bmatrix} C_{aa} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{bb} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{cc} \end{bmatrix} F \tag{10}$$

The off-diagonal terms are negative because a positive voltage applied to a phase induces positive charges on the capacitor of that phase and negative charges on the capacitors of the other phases.

For a balanced capacitor bank, the self and mutual capacitances are [3]

$$C_{aa} = C_{bb} = C_{cc} = C_s \tag{11}$$

$$C_{ab} = C_{ac} = C_{bc} = C_m \tag{12}$$

so that

$$C_{abc} = \begin{bmatrix} C_s & -C_m & -C_m \\ -C_m & C_s & -C_m \\ -C_m & -C_m & C_s \end{bmatrix} \tag{13}$$

Applying similarity (modal) transformation results in

$$C_{012} = A^{-1}C_{abc}A = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \tag{14}$$

where the zero- and positive-sequence capacitances are given by

$$C_0 = C_s - 2C_m \tag{15}$$

$$C_1 = C_2 = C_s + C_m = C_0 + 3C_m \tag{16}$$

3.3. Series capacitor banks modelling

The phase and sequence impedance matrices for a series capacitor bank are given by [3]

$$Z_{abc} = \begin{bmatrix} -jX_C & 0 & 0 \\ 0 & -jX_C & 0 \\ 0 & 0 & -jX_C \end{bmatrix} \Omega \tag{17}$$

3.4. Series capacitor banks modelling

The basic three-phase two winding transformer is shown in figure 3. Its primitive network, on the assumption that the flux paths are symmetrically distributed between all windings, is represented by the equation

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & y'_m & y'_m & -y_m & y''_m & y''_m \\ y'_m & y_p & y'_m & y''_m & -y_m & y''_m \\ y'_m & y'_m & y_p & y''_m & y''_m & -y_m \\ -y_m & y''_m & y''_m & y_s & y'''_m & y'''_m \\ y''_m & -y_m & y''_m & y'''_m & y_s & y'''_m \\ y''_m & y''_m & -y_m & y'''_m & y'''_m & y_s \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} \qquad (18)$$

where y'_m is the mutual admittance between primary coils. y''_m is the mutual admittance between primary and secondary coils on different cores and y'''_m is the mutual admittance between secondary coils [1].

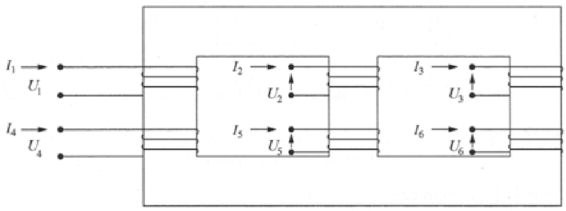

Fig. 3 Diagramatic representation of a two-winding transformer

If a tertiary winding is also present, the primitive network consists of nine (instead of six) coupled coils and its mathematical model will be a 9x9 admittance matrix.

The interphase coupling can usually be ignored (e.g. the case of three single-phase separate units) and all the primed terms are effectively zero.

The connection matrix $[A]$ between the primitive network and the actual transformer buses is derived from the transformer connection.

By way of example, consider the Wye G-Delta connection of figure 4. The following connection matrix applies:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_P^a \\ U_P^b \\ U_P^c \\ U_s^A \\ U_s^B \\ U_s^C \end{bmatrix} \qquad (19)$$

or

$$[U_{branch}] = [A][U_{node}] \qquad (20)$$

It can be also written

$$[Y_{node}] = [A]^T [Y_{prim}][A] \qquad (21)$$

And using $[Y_{prim}]$ from equation (18)

$$[Y_{node}] =$$

$$\begin{array}{cc} \begin{bmatrix} y_p & y'_m & y'_m & -(y_m + y''_m) & (y_m + y''_m) & 0 \\ y'_m & y_p & y'_m & 0 & -(y_m + y''_m) & (y_m + y''_m) \\ y'_m & y'_m & y_p & (y_m + y''_m) & 0 & -(y_m + y''_m) \\ -(y_m + y''_m) & 0 & (y_m + y''_m) & 2(y_s - y'''_m) & -(y_s - y'''_m) & -(y_s - y'''_m) \\ (y_m + y''_m) & -(y_m + y''_m) & 0 & -(y_s - y'''_m) & 2(y_s - y'''_m) & -(y_s - y'''_m) \\ 0 & (y_m + y''_m) & -(y_m + y''_m) & -(y_s - y'''_m) & -(y_s - y'''_m) & 2(y_s - y'''_m) \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ A \\ B \\ C \end{matrix} \end{array} \qquad (22)$$

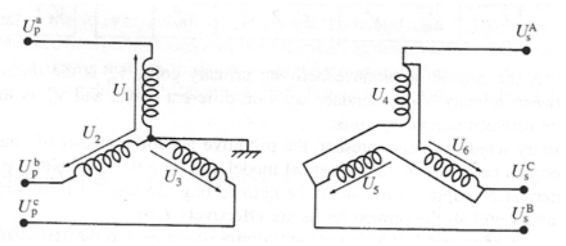

Fig. 4 Network connection diagram for a Wye G-Delta transformer

If the primitive admittances are expressed in per unit the upper-right and lower-left quadrants of matrix (22) must be divided by $\sqrt{3}$ and the lower-right quadrant by 3. Then, in absence of interphase coupling, the nodal admittance matrix of Wye G-Delta connection becomes

$$\begin{bmatrix} I_p^a \\ I_p^b \\ I_p^c \\ I_s^A \\ I_s^B \\ I_s^C \end{bmatrix} = \begin{bmatrix} y & 0 & 0 & -y/\sqrt{3} & y/\sqrt{3} & 0 \\ 0 & y & 0 & 0 & -y/\sqrt{3} & y/\sqrt{3} \\ 0 & 0 & y & y/\sqrt{3} & 0 & -y/\sqrt{3} \\ -y/\sqrt{3} & 0 & y/\sqrt{3} & 2/3y & -1/3y & -1/3y \\ y/\sqrt{3} & -y/\sqrt{3} & 0 & -1/3y & 2/3y & -1/3y \\ 0 & y/\sqrt{3} & -y/\sqrt{3} & -1/3y & -1/3y & 2/3y \end{bmatrix} \begin{bmatrix} U_P^a \\ U_P^b \\ U_P^c \\ U_s^A \\ U_s^B \\ U_s^C \end{bmatrix} \qquad (23)$$

where y is the transformer leakage admittance in per unit [2].

The sequence admittance matrix Y_{012} for Ygd1 transformer can be calculated as $Y_{012} = A^{-1}YA$ and relates the currents and voltages as [3]:

$$\begin{bmatrix} I_{p0} \\ I_{p1} \\ I_{p2} \\ I_{s0} \\ I_{s1} \\ I_{s2} \end{bmatrix} = y \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & e & 0 \\ 0 & 0 & 1 & 0 & 0 & f \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f & 0 & 0 & 1 & 0 \\ 0 & 0 & e & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{p0} \\ U_{p1} \\ U_{p2} \\ U_{s0} \\ U_{s1} \\ U_{s2} \end{bmatrix} \qquad (24)$$

where $e = -1 < 30°$, $f = -1 < 30°$ and

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & a^2 & a & 0 & 0 & 0 \\ 1 & a & a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & a^2 & a \\ 0 & 0 & 0 & 1 & a & a^2 \end{bmatrix}, \quad \begin{matrix} a = 1 < 120^o \\ a^2 = 1 < -120^o \end{matrix} \qquad (25)$$

4. CONCLUSION

Example in chapter 2 shows the differences between current flows on different harmonic order. However the network is very simple it is clear that for example 3rd harmonic (which is zero-sequence harmonic) current flows depends on three-phase elements connection. By the same example can be applied an unsymmetrical voltage source for understanding three-phase elements connection dependence on unsymmetrical currents flows.

This article was describing one way how to model and calculate the three-phase networks. This way was based on phase calculations, but there are more ways how to model and calculate three-phase systems. One often describing method is based on modelling and calculating three-phase networks by symmetrical components obtained by Fortescue method, but in this way it is necessary to solve three different networks, which respects positive-, negative- and zero-sequence flows.

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BIOGRAPHIES

Martin Kanálik was born on 2.11.1981. In 2005 he graduated (MSc.) with distinction at the department of Electric Power Engineering of the Faculty of Electrical Engineering and Informatics at Technical University in Košice. Since 2005 he is studying as a PhD student at the Department Electric Power Engineering. His thesis title is „Decreasing of negative effects of enormous power quality“.

Michal Kolcun was born in 1954. He graduated (MSc.) at the Faculty of Power Engineering at the Moscow Power Engineering Institute in 1979, where he received a PhD in 1989. He became an Associate Professor of Electric Power Engineering at the Faculty of Electrical Engineering and Informatics at the Technical University of Košice. He was promoted as a professor of power and electric engineering in 2000. Since 1979 he has been working at the Faculty of Electrical Engineering and Informatics at the Technical University of Košice.

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