ROBUST CONTROL DESIGN OF A DC DRIVE

Želmíra FERKOVÁ, Ladislav ZBORAY
Department of electrotechnic, mechatronic & industrial engineering
Technical university of Košice, Letná 9, 04 200 Košice.
e-mail: zelmira.ferkova@tuke.sk

SUMMARY

Pole region assignment method ensures sufficient damping for systems with varying poles within the defined interval. It connects analytical and graphical approach for selection of two controller parameters. The same result may be reached by the introduced numerical procedure for a third order control loop. Robust speed controller of a DC drive is designed and analysed. Results are illustrated by time responses obtained by simulation.

Keywords: *robust, state control , controller design, DC drive*

1. INTRODUCTION

Pole region assignment method derived by Ackermann, Kaesbauer and Muench [1] ensures placement of all varying poles of the system characteristic polynomial in the *gamma region* (Fig.1). Two constant feedback parameters providing the desired time responses may be determined. Since the state speed controller generally needs four parameters, system order reduction is recommended. If the superimposed speed loop is satisfactorily slower than the inner current loop, singular perturbation method [4] decreases the order of speed equation to two. However, this condition is often not fulfilled in high-dynamic electrical drives, therefore modified version [6] reduces the equation order to three. Then one controller parameter (usually integrator gain) is chosen and two feedbacks are calculated. This method was successfully applied for DC drive [7] as shown below. Robust control of an induction motor with varying main inductance and moment of inertia was realized and verified by measurement in laboratory [3]. The aim of this paper is oriented to numerical design of a robust speed controller for DC drive and analysis of its properties.

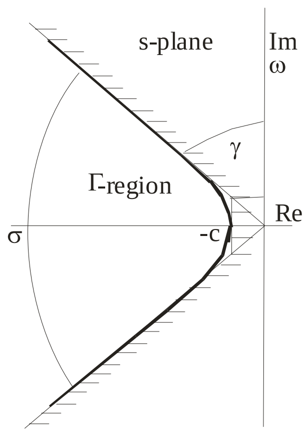

Fig. 1 Definition of the Γ -region

2. POLE REGION ASSIGNMENT METHOD

Poles of the drive characteristic polynomial with varying parameters should never abandon the section determined by the required value of damping $d = \sin\gamma$ and a certain distance $-c$ from the imaginary axis. These conditions define a broken-line form of the region boundary therefore is advantageous continuous description by a hyperbole. Usual damping of electrical drives $d=0.7$ corresponds to 45° angle between imaginary axis and the left branch of a *rectangular* hyperbole (Fig.1). Its equation is

$$\omega^2 = \sigma^2 - c^2 \, . \tag{1}$$

The aim of control is to find such *constant* feedback vector $\boldsymbol{r}$ that ensures placement of varying poles within the Γ -region:

$$u = -\boldsymbol{r}^T \boldsymbol{x} \tag{2}$$

The condition of Γ -stability [1] for a characteristic polynomial with one complex pair of poles $\lambda_{1,2} = \sigma(\alpha) \pm j\omega(\alpha)$ is:

$$\begin{bmatrix} h_0(\alpha) & h_1(\alpha) & \dots & h_n(\alpha) \\ 0 & h_0(\alpha) & \dots & h_{n-1}(\alpha) \end{bmatrix} \boldsymbol{a}(\boldsymbol{p}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

where $h_0(\alpha) = 1$, $h_1(\alpha) = 2\sigma(\alpha)$,

$h_{i+1}(\alpha) = 2\sigma(\alpha)h_i(\alpha) - [\sigma^2(\alpha) + \omega^2(\alpha)]h_{i-1}(\alpha)$

for $i=1,2,\dots n-1$.

$\boldsymbol{a}(\boldsymbol{p})$ is the coefficient vector of the characteristic polynomial containing vector of varying parameters $\boldsymbol{p}$:

$$P(\boldsymbol{p},s) = a_0(\boldsymbol{p}) + a_1(\boldsymbol{p})s + \dots + a_n(\boldsymbol{p})s^n = \\ = [1 \; s \; \dots s^n]\boldsymbol{a}(\boldsymbol{p}) \tag{4}$$

Then σ is substituted by generalized frequency α varying within interval $\alpha \in (-c, -\infty)$ and ω is excluded by hyperbole equation (1):

$$h_0(\alpha)=1,\; h_1(\alpha)=2\alpha,\; h_2(\alpha)=c^2+2\alpha^2,$$

$$h_3(\alpha)=4c^2\alpha$$

Conditional equations now are

$$\begin{bmatrix} 1 & 2\alpha & c^2+2\alpha^2 & 4c^2\alpha & \dots & n_n \\ 0 & 1 & 2\alpha & c^2+2\alpha^2 & \dots & n_{n-1} \end{bmatrix} \boldsymbol{a}(\boldsymbol{p}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (5)$$

Curves for limiting parameters p_{min} and p_{max} are drawn in R-plane dividing it to several areas (Fig.3). The searched solution is found by checking Γ-stability condition for a point chosen within the particular area.

This method is applied for robust speed controller design of a *DC* drive with parameters. *2.3 kW , 220 V,12.3 A, 2800 rev/min., J=0.0315 Ws*^{3}*,* $R_a=1.6\ \Omega$, $T_a=0.01$ *s.* The block scheme with simplified current loop is shown in Fig.2.

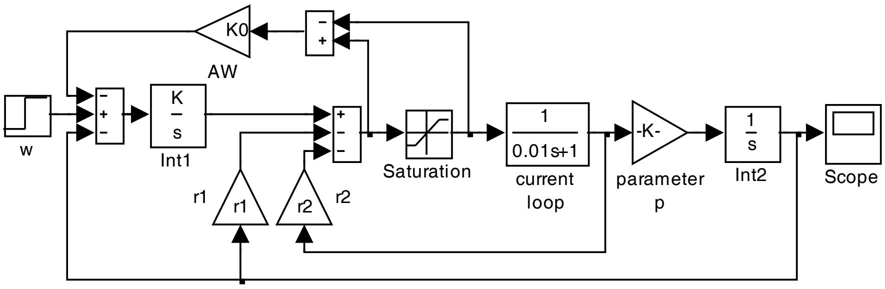

Fig. 2 DC drive with speed state control

Values of hyperbole axes $c=-1$ and integrator gain $K=200$ corresponding to operation with nominal parameters were chosen. Uncertain parameter $p=c\Phi/J$ is expected to vary within interval $p\in(22.2,5.55)$. The maximal value is given by name plate data, the minimal value is chosen for half of the nominal magnetic flux and double moment of inertia. The drive characteristic polynomial is

$$P(\lambda)=\lambda^3+m(1+r_2)\lambda^2+pmr_1\lambda+pmK \quad (6)$$

where $m=1/2T_a=50$. Conditional equations (5) now are

$$\begin{bmatrix} 1 & 2\alpha & c^2+2\alpha^2 & 4c^2\alpha \\ 0 & 1 & 2\alpha & c^2+2\alpha^2 \end{bmatrix} \begin{bmatrix} pmK \\ pmr_1 \\ m(1+r_2) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (7)$$

Two algebraic equations are obtained after multiplication

$$pmK+2pmr_1\alpha+m(1+r_2)(c^2+2\alpha^2)+4c^2\alpha=0$$

$$pmr_1+2m(1+r_2)\alpha+c^2+2\alpha^2=0 \quad (8)$$

Their solution gives feedback parameters expressed as

$$r_1=\frac{2K\alpha}{c^2-2\alpha^2}-\frac{c^2-2\alpha^2}{pm} \quad (9)$$

$$r_2=-1-\frac{2\alpha}{m}-\frac{pK}{c^2-2\alpha^2} \quad (10)$$

Curves for p_{min} varying within interval $\alpha\in(-c,-\infty)$ and for p_{max} within interval $\beta\in(-c,-\infty)$ are drawn in Fig.3. Feedback parameters should be chosen for the operating point in the hatched area, in our case was selected *H* (*25, 4.5).*

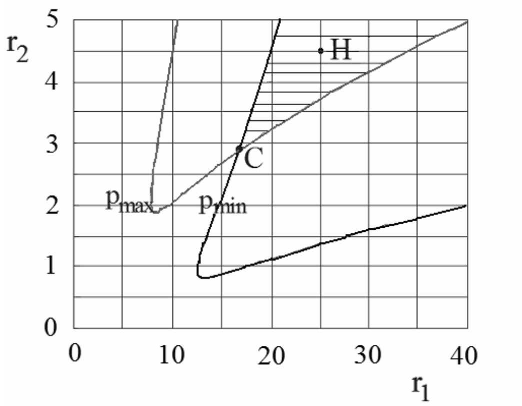

Fig. 3 Choice of the operating point *H* in *R*-plane

Time responses of speed x_1 and armature current x_2 for nominal motor parameter p_{max} (index N) and changed parameter p_{min} are shown in Fig.4.

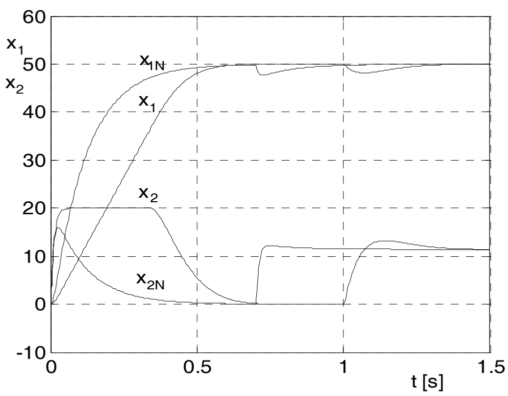

Fig. 4 Speed and current time responses for the chosen operating point H

3. NUMERICAL METHOD

Conditional equations are based on the fact that

- complex pole situated on the Γ-region boundary has the same absolute value of real and imaginary components
- feedback parameters r_1, r_2 are identical for p_{min} and p_{max}.

Characteristic equation of a third order system consists of a complex pole with equal components and one real negative pole

$$s_{1,2} = -a \pm ja\,,\quad s_3 = -d \tag{11}$$

The corresponding polynomial is

$$P(s) = s^3 + (d+2a)s^2 + (2ad+2a^2)s + 2a^2 d \tag{12}$$

Its comparison with the system characteristic polynomial (6) gives

$$d + 2a = m(1 + r_2) \tag{13}$$

$$2ad + 2a^2 = mpr_1 \tag{14}$$

$$2a^2 d = mpK \tag{15}$$

Real pole d expressed from (13) and substituted into (14) and (15) enables to compile expressions for r_1 and r_2:

$$r_1 = \frac{K}{a} + \frac{2a^2}{mp} \tag{16}$$

$$r_2 = -1 + \frac{2a}{m} + \frac{pK}{2a^2} \tag{17}$$

Pole component a corresponds to p_{min}, component b is defined for p_{max} in (16), (17). Then equations determining a, b are derived for equal feedback parameters r_1, r_2 :

$$r_1 \Rightarrow \quad \frac{K}{a} + \frac{2a^2}{mp_{min}} = \frac{K}{b} + \frac{2b^2}{mp_{max}} \tag{18}$$

$$r_2 \Rightarrow \quad -1 + \frac{2a}{m} + \frac{p_{min}K}{2a^2} = -1 + \frac{2b}{m} + \frac{p_{max}K}{2b^2} \tag{19}$$

Solution of this set presents absolute pole values *a=12.782, b=89.715*. Equations (16) and (17) determine feedback parameters

$$r_1 = 16.731, \quad r_2 = 2.864.$$

Third poles follow from (13). Thus poles are

for p_{min} $\quad s_{1,2} = -12.782 \pm j12.782,\ \ s_3 = -167.636$

for p_{max} $\quad s_{1,2} = -89.715 \pm j89.715\,,\ \ s_3 = -13.77$

Both real and imaginary values of complex poles are equal what confirms their position on the Γ-region boundary.

Root locus trajectories (Fig.5) show existence of two operating points p_{min} and p_{max} fulfilling the above defined conditions. With increasing p the real pole λ_1 moves to right and complex poles λ_2, λ_3 are getting near to the multiple point ($p \doteq 10$). For $p \in (10, 21)$ all poles become real. Further increasing of $p > 21$ causes that poles λ_1 and λ_2 create a complex pair and λ_3 remains real.

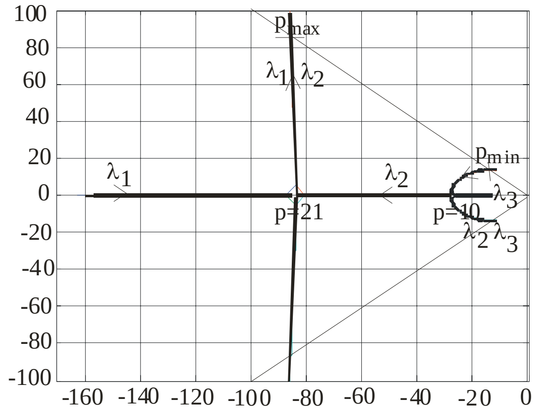

Fig. 5 Root locus with varying parameter *p*.

Speed and current time responses for p_{min} (Fig.6) corresponding to the pair of dominant poles and one real pole situated far to left produce the required form of time response with a small overshoot. Speed time response for p_{max} (index *N*) is characterized by higher damping because the real pole λ_3 is situated nearer to the imaginary axis and becomes dominant.

Since the operating point *H* in Fig.3 was chosen near to the intersection point *C*, time responses of Fig.4 and Fig 6 are similar. Current value at p_{min} must be higher because magnetic flux equals to half of the nominal value. Therefore it was limited at starting-up. The load torque at nominal parameters (p_{max}) was decreased to the halved value for to compare both responses.

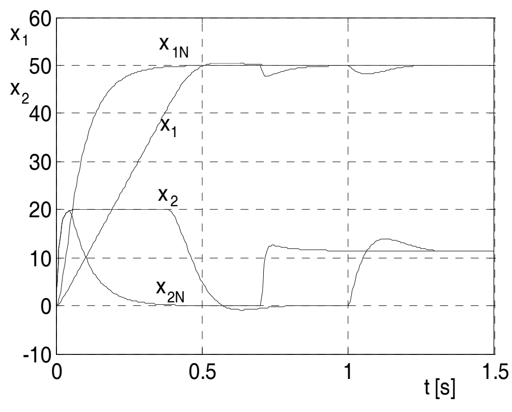

Fig. 6 Speed and current time responses

The above introduced consideration assumes properly chosen integrator gain K. Equations (18) and (19) were repeatedly calculated with different values of K for to evaluate its influence on feedback parameters r_1, r_2 . Results introduced in *Tab.1* show that operating point C in Fig.3 is shifted to higher gain values but the desired interval of robustness may be reached. Minimal K is limited by requirement of positive feedback parameters.

K	a	b	r_1	r_2
200	12.78	89.71	16.73	2.86
300	14.70	102.66	21.96	3.44
400	16.23	113	26.54	3.87
500	17.48	121.72	30.80	4.24
600	18.58	129.34	34.78	4.57

Tab. 1 Influence of estimated integrator gain *K*

Pole assignment method recommends choice of the real pole at least two times more negative than the complex pole components. Thus

$$s_{1,2} = -a \pm ja \,, \quad s_3 = -2a \,.$$

Corresponding polynomial is

$$P(s) = s^3 + 4as^2 + 6a^2 s + 4a^3$$

Comparison with the system characteristic polynomial (6) gives following expressions:

$$r_1 = \frac{6a^2}{mp}\,, \quad r_2 = \frac{4a}{m} - 1\,, \quad K = \frac{4a^3}{mp} \qquad (20)$$

As expected, controller parameters r_1 and K are indirectly proportional to *p.*

Since r_2 does not depend on p (20), certain value of pole $a = m/4$ may be selected at which $r_2 = 0$. Then parameters calculated for p_{min} are $r_1 = 3.36$, $K = 56.3$. Settling time $t_s = 0.265s$ determined by equations (11), (12) provides a little slower time responses (Fig.7) but robust properties within the defined interval remain.

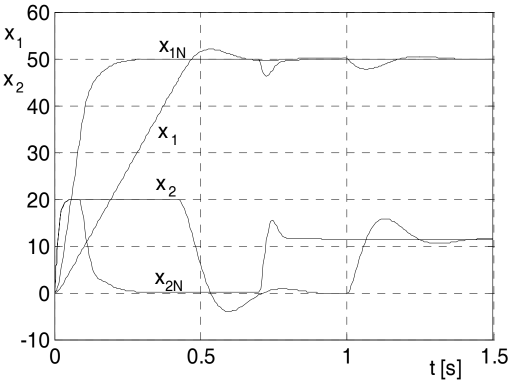

Fig. 7 Time responses at feedback gain $r_2 = 0$.

4. CONCLUSION

The pole region assignment method may be advantageously applied for controller design in electrical drives with cascade structure. Though it enables to determine only two feedback parameters, this approach offers satisfactory solution after system order reduction. Constant feedback parameters ensure insensibility to parameter variation in the defined interval with acceptable changing of damping.

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BIOGRAPHIES

Želmíra Ferková (Doc. Ing. PhD.) graduated in electrical engineering from the Technical University Košice in 1982 and received her PhD in 1994. At present she is active as associated professor of electric machines at the Department of electrical, mechatronic and industrial engineering TU Košice. Her professional area is design of electrical machines , mainly of switched reluctance motor and its performance.

Ladislav Zboray (Prof. Ing. CSc.) received the degree of Ing. in electrical engineering from the Slovak Technical University Bratislava in 1953 and CSc. (PhD) from the University of Transport and Telecommunication Žilina in 1964. After a short industrial practice he has been with Technical University of Košice, since 1982 as professor at the Department of electrical, mechatronic and industrial engineering, at present retired. His major field of interest is the control of electrical drives.