

## SPEED CONTROL BY BACKSTEPPING WITH NONLINEAR OBSERVER OF A PERMANENT MAGNET SYNCHRONOUS MOTOR

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### SUMMARY

*Electrical action of higher performance of permanent magnets synchronous motors requires advanced control strategies related to complex dynamic behaviour of machines.*

*Most of these machines are not linear and are characterised by structural or non structural uncertainties varying with time which make their control very complicated to be implemented.*

*To solve this problem many approaches have been developed. The backstepping control allows to guarantee the robustness of uncertain and disturbed systems by attenuation of the effects of external disturbances to a desired level.*

*This paper is intended to put into evidence the control behaviour based on this new technique.*

*This will be applied to permanent magnets synchronous machine.*

**Keywords:** backstepping nonlinear control (BNC), Feedback state Nonlinear Control (NLC), permanent magnets synchronous motor (PSMS), Lyapunov control function.

### 1. INTRODUCTION

During many years, industries have used direct current motor which has the advantage to be easily controlled by means of natural decoupling of the flux and torque for variable transmission.

However the presence of the collector had been a major disadvantage among other which limits its use [1]. Facing this limitation, the permanent magnets synchronous motors are more attractive because they meet all the requirements.

Also, the coupling that exists between the flux and the torque make this (PSMS) difficult to control. With the non linear control it is possible to obtain good speed variator performance with AC current as those with DC current [2] when the, controlled part is submitted to disturbances and to varying systems parameters; an auto adaptive solution has to be sleeked which by readjustment of the rectifiers parameters, allows to conserve the predefined performances.

This solution has the disadvantage of requiring a complex implementation. Thus it is possible to try other simple solution which will recall a particular class of control systems "referred as" backstepping linear.

The main interest in this type of control is the simplicity of the solution to the problems related to the construction of a good quality control system for process with variable parameters.

This new technique permits to put into evidence the simplicity of the design and higher performances obtained compared to those obtained by classical control methods (by poles placement).

The simulation result obtained are compared between the two control systems developed in this study for speed control in the same operating conditions (under load, disturbances).

### 2. NONLINEAR MODEL OF THE PSMS

The model of the PSMS is expressed with respect to the reference related the rotor under the form of a state and, with simplifying hypothesis [1] [2] [3] [4] as:

$$\begin{aligned} \dot{x} &= f(x) + gu \\ y &= h(x) \end{aligned} \quad (1)$$

$$\text{With: } x = [x_1 \quad x_2 \quad x_3] = [i_d \quad i_q \quad \Omega_r]$$

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \begin{pmatrix} -\frac{R}{L_d} x_1 + p \frac{L_q}{L_d} x_2 x_3 \\ -\frac{R}{L_q} x_2 - p \frac{L_d}{L_q} x_1 x_3 - p \frac{\phi_f}{L_q} x_3 \\ -\frac{B}{J} x_3 + p \frac{(L_d - L_q)}{J} x_1 x_2 + p \frac{\phi_f}{J} x_2 - \frac{T_L}{J} \end{pmatrix}$$

The controlled variables are the current  $x_1 = i_d$  and the mechanical speed  $x_3 = \Omega_r$ .

$$y(x) = \begin{pmatrix} h_1 \\ h_3 \end{pmatrix} = \begin{pmatrix} i_d \\ \Omega_r \end{pmatrix}; g = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} = \begin{bmatrix} 1/L_d & 0 \\ 0 & 1/L_q \end{bmatrix} \quad (2)$$

where,

$u = (u_d \quad u_q)^T$  = voltage vector:

$u_d, u_q$  = d and q-axis stator voltages;

$L_d, L_q$  = d and q-axis stator inductances;

$T_e, T_L$  = electromagnetic and load torques;

$i_d, i_q$  = d and q-axis stator currents;

$R$  = stator per phase resistance;

$J$  = moment of inertia of the motor and load;

$B$  = friction coefficient of the motor;

$p$  = number of poles of the motor;

$\Omega_r$  = rotor speed in angular frequency;

$\phi_f$  = rotor magnetic flux linking the stator.

### 3. INPUT-OUTPUT LINEARISATION

The linearization condition which permits to verify if a system is nonlinear admits an input – output linearization is the order of the relative degree of the system [1][2][3][4].

#### 1– Relative Degree

The output relative degree is the number of times to derive in order to make the input  $u$  appear.

**a – relative Degree of current  $i_d$  :**

$$\dot{y}_1(x) = L_f h_1(x) + L_g h_1(x) u_d \quad (3)$$

$$\text{With } \begin{cases} L_f h_1(x) = f_1(x) \\ L_g h_1(x) = (g_1 \ 0) \end{cases}$$

The relative degree of  $y_1(x)$  is  $r_1=1$ .

**b – Relative Degree of mechanical speed  $\Omega$**

$$\dot{y}_2(x) = L_f h_2(x) \quad (4)$$

$$\ddot{y}_2(x) = L_f^2 h_2(x) + L_g L_f h_2(x) u_q$$

With

$$L_f h_2(x) = f_3(x)$$

$$L_f^2 h_2(x) = c_2 x_2 f_1(x) + f_2(x)(c_3 + c_2 x_1) + c_1 f_3(x)$$

$$L_g L_f h_2(x) = [c_2 x_2 g_1 \quad g_2(c_2 x_1 + c_3)]$$

$$\text{With: } c_1 = \frac{-B}{J}, \quad c_2 = \frac{p(L_d - L_q)}{J}, \quad c_3 = \frac{p\phi_f}{J}$$

The relative degree  $y_2(x)$  is  $r_2=2$

The total degree of the system is equal to the order  $n$  ( $r = r_1 + r_2 = n = 3$ ). The system can be linearised. The matrix which defines the relation between the physical input  $u$  and the output derivatives  $y(x)$  is given by expression (5).

$$\begin{pmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{pmatrix} = \begin{pmatrix} \frac{d}{dt} i_d \\ \frac{d^2}{dt^2} \Omega_r \end{pmatrix} = A(x) + D(x) \begin{pmatrix} u_d \\ u_q \end{pmatrix} \quad (5)$$

With

$$A(x) = \begin{pmatrix} f_1(x) \\ c_2 x_2 f_1(x) + f_2(x)(c_3 + c_2 x_1) + c_1 f_3(x) \end{pmatrix}$$

$$D(x) = \begin{pmatrix} g_1 & 0 \\ c_2 x_2 g_1 & g_2(c_2 x_1 + c_3) \end{pmatrix}$$

#### 2 – Model linearization

To linearise the input–output behaviour of the machine in closed loop, we apply the nonlinear feed back [7]:

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = D^{-1}(x) \left[ -A(x) + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \right] \quad (6)$$

The determinant of the decoupling matrix  $D^{-1}(x)$  different from zero (machine with permanent magnets). The application of the linearization law (6) on the system (5). Leads to two decoupling linear subsystems.

$$\begin{pmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{pmatrix} = \begin{pmatrix} \frac{d}{dt} i_d \\ \frac{d^2}{dt^2} \Omega_r \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (7)$$

### 4. NONLINEAR CONTROL BY POLES PLACEMENT

#### 1– Internal law of control

The internal inputs ( $v_1 \ v_2$ ) are calculated by

imposing a static regime: ( $\dot{i}_{dref} = \dot{i}_d$  et  $\Omega_{ref} = \Omega_r$ ), and a dynamic on the error.

$$\begin{aligned} \frac{d}{dt} e_1 + k_{11} e_1 &= 0 \\ \frac{d^2}{dt^2} e_2 + k_{21} \frac{d}{dt} e_2 + k_{22} e_2 &= 0 \end{aligned} \quad (8)$$

$$\text{With : } \begin{cases} e_1 = i_{dref} - i_d \\ e_2 = \Omega_{ref} - \Omega_r \end{cases}$$

The internal inputs ( $v_1 \ v_2$ ) are defined as:

$$v_1 = k_{11} (i_{dref} - i_d) + \frac{d}{dt} i_{dref} \quad (9a)$$

$$v_2 = k_{21} \left( \frac{d}{dt} \Omega_{ref} - \frac{d}{dt} \Omega_r \right) + k_{22} (\Omega_{ref} - \Omega_r) + \frac{d^2}{dt^2} \Omega_{ref} \quad (9b)$$

$$\text{With } \dot{i}_{dref} = \dot{\Omega}_{ref} = \ddot{\Omega}_{ref} = 0$$

The coefficients  $k_{11}, k_{21}, k_{22}$  are chosen such that equation (10) is d'Hurwitz polynomial.

$$\begin{aligned} p + k_{11} &= 0 \\ p^2 + k_{21}p + k_{22} &= 0 \end{aligned} \quad (10)$$

#### 2 – Physical control Law:

This nonlinear control Law intervenes in the voltage vector:  $u = (u_d \ u_q)^T$ .

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = D^{-1}(x) \left[ -A(x) \begin{pmatrix} k_{11}(i_{dref} - i_d) \\ -k_{21} f_3(x) + k_{22} (\Omega_{ref} - \Omega_r) \end{pmatrix} \right] \quad (11)$$

$f_3(x)$  Intervenes in the component  $u_q$  with the torque of load which is not easily measurable.

The algorithm of the nonlinear control by placement of poles depends excessively on the machine parameters.

During the operating conditions the parameters undergo important variations to diverse phenomena such as temperature variation and magnetic material saturation these variations have effects on the machine and make it loose its required tuning quality.

This has led to search for an adequate control which is backstepping non linear control. This technique which is adopted in the present study

allows, eliminating all divergences between the response of the model and the response of the system for any input and disturbance signals. It achieves appreciable improvements in the performance of the system.

## 5. DESIGN OF BACKSTEPPING CONTROL

This part deals with the speed control strategy by means of backstepping control a new technique of nonlinear control that has been recently developed. It can be applied efficiently to linearise a non linear system with the existence of uncertainties. Compared to the feed back linearization methods [5] [8].

The backstepping nonlinear control technique has robust properties and it is important to exploit them for PMSM control.

The speed track up is made with high efficiency by the voltage control  $u_q$  as long as the current  $i_d$  is maintained equal to zero.

It has been already demonstrated that the control of the PSMS belongs to a class of nonlinear system for which the integration of the backstepping technique can be efficiently used.

This technique is designed to overcome the uncertainties of the system modelling .

It is efficient for linearization of nonlinear system with uncertainties.

The essence of backstepping is the identification of a virtual state control for developing a stabilizing function. At each step of the backstepping a new Lyapunov control function is constructed by the increase of the preceding function by means of a limit that penalizes the error between a variable state and its desired value [8].

The main advantage of the backstepping is the construction of Lyapunov's derivative function which can be made negative and definite through a variety of control laws instead of a specific control law.

This technique guarantees the asymptotic stability of the system in closed – loop. The choice of the virtual control is made after the stabilisation of errors on zero. As the speed error  $e$  must be reduced to zero the components of the current  $i_d$  and  $i_q$  are identified as virtual elements of control for stabilizing the speed of the motor.

The dynamic error of the speed is

$$\dot{e} = -J\dot{\Omega}_r = B\Omega_r + T_L - p(\phi_f i_q + (L_d - L_q)i_d i_q) \quad (12)$$

Where  $e = \Omega_r^* - \Omega_r$  The speed error

In order to determine the stabilizing function Lyapunov's function is defined as:

$$V = \frac{1}{2} e^2 \quad (13)$$

By deriving (13) we obtain:

$$\begin{aligned} \dot{V} = \dot{e}e &= (B\Omega_r + T_L) \frac{e}{J} - \frac{p}{J} (\phi_f i_q + (L_d - L_q)i_d i_q) e \\ &- k_s e^2 + \frac{e}{J} (B\Omega_r + T_L - p\phi_f i_q + k_s J e) \\ &- \frac{p}{J} (L_d - L_q) i_d i_q e \end{aligned} \quad (14)$$

Where  $k_s$  is feedback constant.

The tracking of the speed can be made by the stabilizing function is defined:

$$i_q^* = \frac{1}{p\phi_f} (B\Omega_r + T_L + k_s J e) \quad (15)$$

$$i_d^* = 0 \quad (16)$$

The choice to be made to guarantee the attraction of the controlled variable toward its reference value

$e_d = i_d^* - i_d$  and  $e_q = i_q^* - i_q$  and thus construct

control such as the derivative of Lyapunov's function (13) becomes:

$$\dot{V} = -k_s e^2 \quad (17)$$

From this fact the global stability can be achieved and appropriate estimate must be made in adaptive manner the torque  $T_L$  which is an unknown, we define:

$$\hat{T}_L^* = \frac{1}{p\phi_f} (B\Omega_r + \hat{T}_L + k_s J e) \quad (18)$$

Where  $\hat{T}_L$  is the estimated value of the torque.

From (12) and (18) the dynamic error is obtained:

$$\dot{e} = \frac{1}{J} (\tilde{T}_L + p\phi_f e_q - k_s e J + p(L_d - L_q)e_d i_q) \quad (19)$$

where :  $\tilde{T}_L = \hat{T}_L - T_L$

To stabilise the current component  $i_d$  et  $i_q$  we define there dynamic errors along the axes d and q:

$$\dot{e}_d = i_d^* - \dot{i}_d = \frac{R}{L_d} i_d - \frac{p\Omega_r L_q}{L_d} i_q - \frac{u_d}{L_d} \quad (20)$$

$$\begin{aligned} \dot{e}_q = i_q^* - \dot{i}_q &= \frac{1}{p\phi_f} (B \frac{d\Omega_r}{dt} + k_s J \frac{de}{dt}) - \frac{di_q}{dt} = \\ &\frac{1}{p\phi_f} (B - k_s J) \frac{d\Omega_r}{dt} - (-\frac{R}{L_q} i_q - \frac{p\Omega_r L_d}{L_q} i_d + \frac{u_q}{L_q} - \frac{p\Omega_r \phi_f}{L_q}) - \\ &\frac{(B - k_s J)}{p\phi_f J} (p(\phi_f i_q + (L_d - L_q)i_d i_q) - B\Omega_r - T_L) \\ &+ \frac{Ri_q}{L_q} + \frac{p\Omega_r L_d}{L_q} i_d - \frac{u_q}{L_q} + \frac{p\Omega_r \phi_f}{L_q} \end{aligned} \quad (21)$$

### 1 – Calculation of the control voltage and computation of the adaptive law

The adaptive law must guarantee the stability and convergence toward zero of the dynamic error.

We choose the adaptive law such that the derivative of this function with respect to time is negative Lyapunov's function represents the sum of the quadratic forms of dispersion and adaptation [6] [5].

A new Lyapunov's function can be defined including the error variables  $e_d$  and  $e_q$ .

$$V_2 = \frac{1}{2}(e^2 + e_d^2 + e_q^2 + \frac{1}{\gamma}\tilde{T}_L^2) \quad (22)$$

Where  $\gamma$ : is the adaptive gain

We take the derivative with respect to time of  $V_2$ :

$$\begin{aligned} \dot{V}_2 &= \dot{e} + e_d \dot{e}_d + e_q \dot{e}_q \\ \dot{V}_2 &= -k_s e^2 - k_1 e_d^2 - k_2 e_q^2 \\ &\quad + \frac{e}{J}(\tilde{T}_L + p\phi_f e_q - k_s e J + p(L_d - L_q)e_d i_q) \\ &\quad + e_d \left( \frac{R}{L_d} i_d - \frac{p\Omega_r L_q}{L_d} i_q - \frac{u_d}{L_d} + k_1 e_d \right) \\ &\quad + e_q \left( \frac{(B - k_s J)}{p\phi_f J} (p(\phi_f i_q + (L_d - L_q)i_d i_q) - B\Omega_r - T_L) \right. \\ &\quad \left. + \frac{R}{L_q} i_q + \frac{p\Omega_r L_d}{L_q} i_d - \frac{u_q}{L_q} + \frac{p\Omega_r \phi_f}{L_q} + k_2 e_q \right) \\ &\quad + \frac{1}{\gamma} \tilde{T}_L \dot{\tilde{T}}_L \end{aligned} \quad (23)$$

We define the voltage of control along the axis (d, q):

$$u_d = R i_d - p\Omega_r L_q i_q + k_1 e_d L_d + \frac{p}{J} L_d (L_d - L_q) i_q e \quad (24)$$

$$\begin{aligned} u_q &= \frac{L_q (B - k_s J)}{p\phi_f J} (p(\phi_f i_q + (L_d - L_q)i_d i_q) - B\Omega_r - \hat{T}_L) \\ &\quad + R i_q + p\Omega_r L_d i_d + p\Omega_r \phi_f + k_2 e_q L_q + \frac{p}{J} \phi_f e L_q \end{aligned} \quad (25)$$

Finally we obtain the following expression for the equation (23) which becomes after simplification:

$$\begin{aligned} \dot{V}_2 &= \frac{e}{J}(\tilde{T}_L - k_s e J) + \frac{e_d}{L_d}(-k_1 e_d L_d) + \\ &\quad \frac{e_q}{L_d} \left( -\frac{L_q (B - k_s J)}{p\phi_f J} (T_L - \hat{T}_L) - k_2 e_q L_q \right) + \frac{1}{\gamma} \tilde{T}_L \dot{\tilde{T}}_L \end{aligned} \quad (26)$$

from the equation (26) It results:

$$\begin{aligned} \dot{V}_2 &= -k_s e^2 - k_1 e_d^2 - k_2 e_q^2 \\ &\quad + \tilde{T}_L \left( \frac{e}{J} + \frac{e_q (B - k_s J)}{p\phi_f J} + \frac{1}{\gamma} \dot{\tilde{T}}_L \right) \end{aligned} \quad (27)$$

Thus it is sometimes necessary to estimate in an adaptive way the couple of load.

This evaluation can be:

$$\dot{\tilde{T}}_L = -\gamma \left( \frac{e}{J} + \frac{e_q (B - k_s J)}{p\phi_f J} \right) \quad (28)$$

We obtain finally the expression for

$$\dot{V}_2 = -k_s e^2 - k_1 e_d^2 - k_2 e_q^2 \leq 0 \quad (29)$$

## 6. PROPOSED OBSERVER

To avoid the use of linked flux and mechanical-position sensors, it is necessary to estimate the rotor speed and the position derivatives of flux from the electrical measurable variables.

In (32), we can observe that the EMF contains the

necessary information to determine speed and rotor position, which can be obtained by current and voltage measurements. Thus, it is possible to design an observer to estimate EMF from which speed and rotor position can be obtained. In order to do that, the time derivative of EMF needs to be done, obtaining:

$$e_\alpha = \frac{\partial \lambda_\alpha}{\partial t} = \frac{\partial \lambda_\alpha}{\partial \theta} \omega = \phi_\alpha(\theta) \omega \quad (30)$$

$$e_\beta = \frac{\partial \lambda_\beta}{\partial t} = \frac{\partial \lambda_\beta}{\partial \theta} \omega = \phi_\beta(\theta) \omega$$

$$\frac{de_\alpha}{dt} = \frac{\partial \phi_\alpha(\theta)}{\partial \theta} \omega^2 + \phi_\alpha(\theta) \frac{d\omega}{dt} \quad (31)$$

$$\frac{de_\beta}{dt} = \frac{\partial \phi_\beta(\theta)}{\partial \theta} \omega + \phi_\beta \frac{d\omega}{dt}$$

As it can be seen in (30), the information about rotor speed and position derivatives of flux (f functions) can be obtained from the induced EMF [7][9]. To do that, a novel nonlinear observer is proposed in this paper. This proposal uses a nonlinear reduced-order observer and a high-gain observer to estimate the induced EMF without previous knowledge of its waveform, measuring motor currents and voltages. Once the EMF is estimated, the position derivatives of the linked flux and the rotor speed are calculated.

The time derivatives of the induced EMF components  $\alpha$ ,  $\beta$  in (30) are first calculated to implement the observer. The PSMS dynamic model, in a stationary reference frame  $\alpha$ - $\beta$ , can be presented as follows [7] [9]:

$$\frac{di_\alpha}{dt} = -\frac{R}{L} i_\alpha - \frac{1}{L} e_\alpha + \frac{1}{L} v_\alpha \quad (32)$$

$$\frac{di_\beta}{dt} = -\frac{R}{L} i_\beta - \frac{1}{L} e_\beta + \frac{1}{L} v_\beta$$

$$\begin{cases} \frac{d\theta}{dt} = \omega \\ \frac{d\omega}{dt} = \frac{1}{J} T_e - \frac{B}{J} \omega \end{cases} \quad \text{et } \Omega = \frac{\omega}{p} \quad (33)$$

Where  $i_\alpha$ ,  $i_\beta$ ,  $e_\alpha$ ,  $e_\beta$ ,  $v_\alpha$ , and  $v_\beta$  represent the current, induced EMF and voltage components respectively, in a stationary reference frame. The variables  $\theta$ ,  $\omega$ ,  $T_e$  and  $T_L$  represent the rotor position and speed, the electromagnetic torque produced by the motor and the load torque, respectively.

From (31), the following EMF observer can be designed:

$$\begin{aligned} \frac{d\hat{e}_\alpha}{dt} &= \frac{\partial \hat{\phi}_\alpha(\theta)}{\partial \theta} \hat{\omega}^2 + \hat{\phi}_\alpha(\theta) \frac{d\hat{\omega}}{dt} + g \left( L \frac{d\hat{i}_\alpha}{dt} - L \frac{di_\alpha}{dt} \right) \\ \frac{d\hat{e}_\beta}{dt} &= \frac{\partial \hat{\phi}_\beta(\theta)}{\partial \theta} \hat{\omega}^2 + \hat{\phi}_\beta(\theta) \frac{d\hat{\omega}}{dt} + g \left( L \frac{d\hat{i}_\beta}{dt} - L \frac{di_\beta}{dt} \right) \end{aligned} \quad (34)$$

Where the time derivatives of stator currents are used as correction terms. An appropriate gain  $g$  must be chosen when adjusting the observer in order to obtain exponential convergence as shown in [7][9].

The estimated current derivatives, necessary for obtaining the correction term, can be obtained from (32),

$$\begin{aligned} \frac{d\hat{i}_\alpha}{dt} &= -\frac{R}{L} i_\alpha - \frac{1}{L} \hat{e}_\alpha + \frac{1}{L} v_\alpha \\ \frac{d\hat{i}_\beta}{dt} &= -\frac{R}{L} i_\beta - \frac{1}{L} \hat{e}_\beta + \frac{1}{L} v_\beta \end{aligned} \quad (35)$$

Furthermore, the time derivative of speed can be obtained from (33),

$$J \frac{d\hat{\omega}}{dt} = T_e(\hat{\theta}, i_\alpha, i_\beta) - B\hat{\omega} \quad (36)$$

If the electromagnetic torque is given by:

$$T_e = \phi_\alpha i_\alpha + \phi_\beta i_\beta \quad (37)$$

Then,

$$\frac{d\hat{\omega}}{dt} = \frac{1}{J} (\hat{\phi}_\alpha i_\alpha + \hat{\phi}_\beta i_\beta) - \frac{B}{J} \hat{\omega} \quad (38)$$

A variable change can be made:

$$\begin{cases} \hat{v}_\alpha = \hat{e}_\alpha + gL i_\alpha \\ \hat{v}_\beta = \hat{e}_\beta + gL i_\beta \end{cases} \quad (39)$$

Making time derivative of (39), and substituting from (34), the proposed observer will result in:

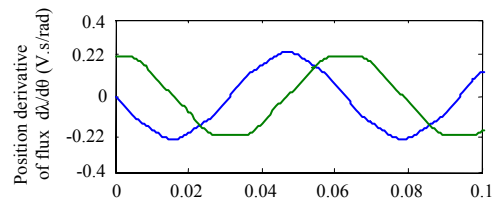
$$\begin{aligned} \frac{d\hat{v}_\alpha}{dt} &= \frac{\partial \phi_\alpha(\theta)}{\partial \theta} \hat{\omega}^2 + \phi_\alpha(\theta) \frac{d\hat{\omega}}{dt} + gL \frac{d\hat{i}_\alpha}{dt} \\ \frac{d\hat{v}_\beta}{dt} &= \frac{\partial \phi_\beta(\theta)}{\partial \theta} \hat{\omega}^2 + \phi_\beta(\theta) \frac{d\hat{\omega}}{dt} + gL \frac{d\hat{i}_\beta}{dt} \end{aligned} \quad (40)$$

The estimated EMF can be obtained from (39). Once the estimated EMF is obtained, the speed and rotor position can be calculated by solving (30).

To design the observer, the time derivative of the linked flux components are changed to  $\alpha$ - $\beta$  coordinates, within a stationary reference frame, thus allowing to obtain the waveforms shown in Fig. 1. Fourier series can be used to approximate these

$$\begin{cases} \phi_\alpha(\hat{\theta}) = \phi_r (-\sin(p\hat{\theta}) - 1/25 \sin(5p\hat{\theta})) \\ \phi_\beta(\hat{\theta}) = \phi_r (-\cos(p\hat{\theta}) - 1/25 \cos(5p\hat{\theta})) \end{cases} \quad (41)$$

Where  $\phi_r$  is the amplitude of the fundamental component and  $p$  is the number of pole pairs.



**Fig. 1** Position derivative of flux, in  $\alpha$ - $\beta$  coordinates (electrical radians).

The position derivatives of these functions, necessary to the implementation of (40), are given by:

$$\begin{cases} \frac{\partial \phi_\alpha(\hat{\theta})}{\partial \theta} = p \phi_r (-\cos(p\hat{\theta}) - 1/5 \cos(5p\hat{\theta})) \\ \frac{\partial \phi_\beta(\hat{\theta})}{\partial \theta} = p \phi_r (-\sin(p\hat{\theta}) - 1/5 \sin(5p\hat{\theta})) \end{cases} \quad (42)$$

Once the induced EMF is estimated, an inverse function of the expressions (42) should be calculated to obtain estimated position. However, it is not possible to calculate it exactly, but a good approximation is given by:

$$\hat{\theta} \approx \frac{1}{p} \tan^{-1} \left( \frac{-e_\alpha / e_\beta}{\phi_\alpha / \phi_\beta} \right) + \frac{1}{25p} \sin \left( 6p \tan^{-1} \left( \frac{-e_\alpha / e_\beta}{\phi_\alpha / \phi_\beta} \right) \right) \quad (43)$$

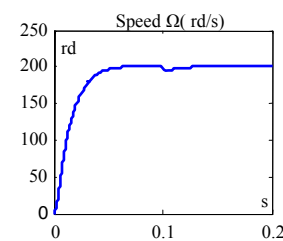
Once the estimated position is obtained, the speed can be calculated, taking into account:

$$\hat{e}_\alpha^2 + \hat{e}_\beta^2 = \hat{\omega}^2 \left[ \phi_\alpha^2(\hat{\theta}) + \phi_\beta^2(\hat{\theta}) \right] \quad (44)$$

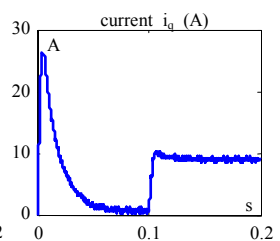
Thus,

$$\hat{\omega} = \sqrt{\frac{\hat{e}_\alpha^2 + \hat{e}_\beta^2}{\phi_\alpha^2(\hat{\theta}) + \phi_\beta^2(\hat{\theta})}} \quad (45)$$

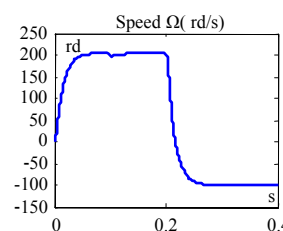
### Feedback state Nonlinear Control NLC



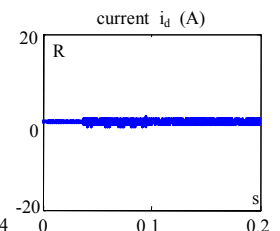
**Fig. 2** Speed response at load torque application



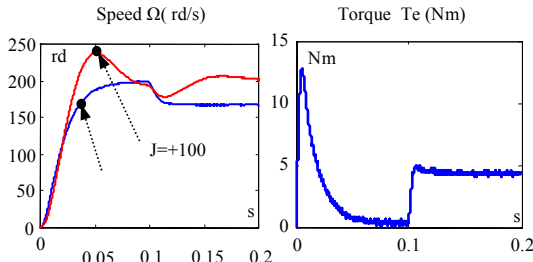
**Fig. 3** Evolution of the current  $i_q$



**Fig. 4** Speed during starting followed by an inversion at 0.2s

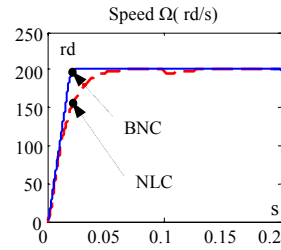


**Fig. 5** Evolution of the current  $i_d$



**Fig. 6** Speed during the variation of - stator resistance à100% - inertia at 100%

**Fig. 7** Evolution of the electromagnetic torque  $T_e$



**Fig. 14** Comparison of the two controls

**7. SIMULATION**

The performances of the proposed control have been tested by numerical simulation for the motor whose parameters and characteristics are given in the appendix.

It is observed from the obtained results fig. 2, 3,4 and 5 that the technique of nonlinear control has a good performance for the disturbances rejection. Also, the technique of nonlinear control is very sensible to any variation of internal parameters and machine disturbance because the algorithm of the control is dependant on machine parameters which are subjected to time variations.

Figure.6 puts into evidence the effect of the variation of the resistance. The variator performance are deteriorated and the simulation results shows clearly this degradation.

In order to put into evidence the advantage of bacstepping control with the respect to the pole placement a numerical simulation has been carried out with a 100% variation of the statoric resistance. The results obtained are shown in figures (8) and (13).

The results (fig.13) show a better robustness of backstepping control compared to that of pole placement on the basis of parametric variation. Figure 14 shows the comparison between the two controls (BNC and NLC).

**8. CONCLUSION**

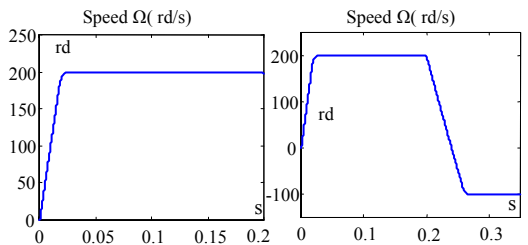
The present analysis has allowed to put into evidence the static and dynamic properties of the backstepping control with nonlinear feedback by comparison of the result obtained for speed control whatever are the operating ranges studied .

The response by backstepping control is fast and robust during disturbances represented by nominal load torque and the parametric variations of the motor.

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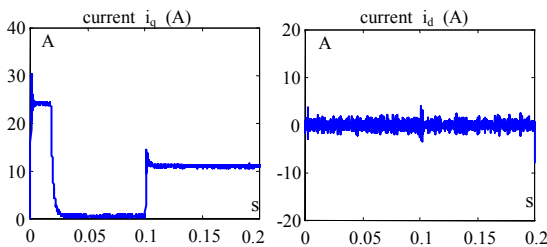
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**Backstepping Nonlinear control (BNC)**



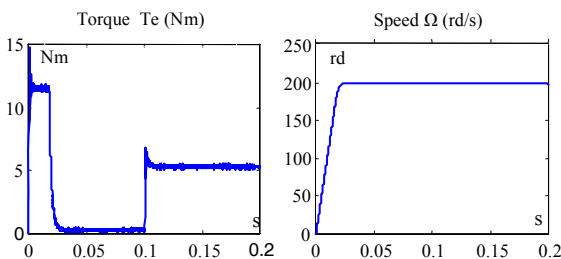
**Fig. 8** Speed during starting, with disturbance rejection

**Fig.9** Speed during starting followed by an inversion at 0.2s



**Fig. 10** Evolution of the current  $i_q$  with limitation

**Fig. 11** Evolution of the current  $i_d$



**Fig. 12** Evolution of the electromagnetic torque  $T_e$

**Fig. 13** Test of robustness during the variations - stator resistance à100%. - inductances  $L_d=50\%$  and  $L_q=50\%$

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