SYNTHESIS OF LOSS EXPONENTIAL TRANSMISSION LINE

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SUMMARY

Loss exponential transmission line as impedance transformer, with capacitance per unit length approximated by power series, is considered in this paper. The values of input impedance transformed through exponential and the considered transmission line are presented. Dependence of input impedance at arbitrary distance along the approximated line, its deviation from exponential function for different frequencies and for different taper ratio is observed. Standing wave ratio at input end is calculated for different length of the line and different taper ratio and the results are presented in tables.

Keywords: *input impedance, loss exponential transmission line, taper ratio, standing wave ratio*

1. INTRODUCTION

The nonuniform lines [1-7] and uniform transmission lines [8] and [9] can be used as impedance transformers. Uniform transmission lines act as impedance transformers depending on the frequency and length of the line.

The paper [1] gives solution of closed-form of the equation for value of arbitrary complex impedance transformed through a length of lossless, nonuniform transmission line with exponential, cosine-squared and parabolic taper. Some results for nonuniform exponential loss transmission line used as impedance transformer are presented in papers [2-4].

The nonuniform lines have the advantage of wide-band impedance matching when used as impedance transformers and larger rejection bandwidths when used as filters [5] and [6].

The paper [9] presents an easily understood analysis and some history of the transmission line transformer. Radio frequency transformers consisting of matched transmission lines of equal length and characteristic impedance are presented in [8].

In this paper the loss exponential transmission line with capacitance per unit length approximated by power series is presented. Deviation of input impedance of the considered transmission line from the input impedance of exponential transmission line along the line for different frequencies and for different taper ratio is presented.

2. INPUT IMPEDANCE OF THE LOSS EXPONENTIAL TRANSMISSION LINE

A nonuniform transmission line shown in Fig. 1 is considered. u_g is the source voltage, Z_g and Z_p are source impedance and load impedance, respectively. Assuming the TEM mode of propagation, the behavior of transmission line is described by Telegraph's equations.

A loss exponential transmission line of length d, as in Fig. 1, is considered. If losses are small and constant than series impedance and shunt admittance per unit length of the line are $Z' = R' + j\omega L'$ and $Y' = j\omega C'$.

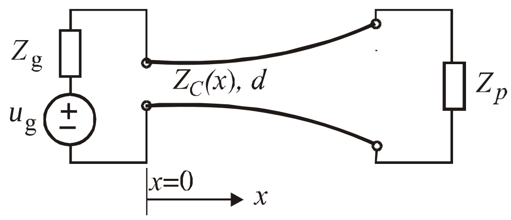

Fig. 1 Schematic presentation of exponential transmission line

Let us assume that the line has following primary parameters $R'=\text{const.}\neq 0$, $G'=0$, $L'=L_0' e^{sx}$ and that the capacitance per unit length, at arbitrary distance x, can be approximated by the power series $C'=C_0'\sum_{i=0}^{n} A_i x^i$, where L_0' and C_0' are inductance and capacitance per unit length at the input end of the exponential line. The constants A_i are chosen to satisfy the condition that taper ratio of the observed transmission line is equal as in the case of exponential taper ratio. s is determined as $s=\frac{1}{d}\ln P$, where P denotes a taper ratio of exponential transmission line which is defined as

$$\mathrm{P} = \mathrm{Z_C}(d)/\mathrm{Z_C}(0). \tag{1}$$

$Z_C(0)$ and $Z_C(d)$ are the characteristic impedances at the source and load ends of line, respectively.

Secondary parameters for considered transmission line are

$$Z_C(x) = Z_C(0)\sqrt{\frac{e^{sx}\left(1 - j\,r\,e^{-sx}\right)}{\sum_{i=0}^{n} A_i x^i}} \tag{2}$$

and

$$\gamma^2(x) = -k_0^2\left(1 - j\,r\,e^{-sx}\right)e^{sx}\sum_{i=0}^{n} A_i x^i , \tag{3}$$

where $r = R'/\omega L_0'$, $k_0 = \omega\sqrt{L_0' C_0'}$ and $Z_C(0) = \sqrt{L_0'/C_0'}$. State on the transmission line can be expressed by the following differential equation

$$U'' - \frac{s\left(1 + j\,r\,e^{-sx}\right)}{1 + r^2 e^{-2sx}} U' + k_0^2\left(1 - j\,r\,e^{-sx}\right)e^{sx}\sum_{i=0}^{n} A_i x^i\, U = 0. \tag{4}$$

The input impedance at the distance x, is determined as

$$Z(x) = \frac{U(x)}{I(x)} . \tag{5}$$

Solving (4) and (5), and satisfying conditions $Z_g = Z_c(x=0)$, $Z_p = Z_c(x=d)$ and $U_g = 2\mathrm{V}$, for $Z(x)$ is obtained

$$Z(x) = Z'(x)\frac{\left[1 + G\,\gamma_2(d)\right] e^{k_2 d} e^{k_1 x} - \left[1 + G\,\gamma_1(d)\right] e^{k_1 d} e^{k_2 x}}{\left[1 + G\,\gamma_1(d)\right] e^{k_1 d}\,\gamma_2(x) e^{k_2 x} - \left[1 + G\,\gamma_2(d)\right] e^{k_2 d}\,\gamma_1(x) e^{k_1 x}} , \tag{6}$$

where $G = \dfrac{Z_C(d)}{Z'(d)}$, $\underline{k}_1(x) = k_{1r} + jk_{1i}$, $\underline{k}_2(x) = k_{2r} + jk_{2i}$

$$k_{1r} = b + \frac{re^{-sx}\left(2b^2 + k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right)}{2\sqrt{-\frac{1}{2}\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right] - \frac{1}{2}\sqrt{\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right]^2 + r^2 e^{-2sx}\left(2b^2 + k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right)^2}}} , \tag{7}$$

$$k_{1i} = be^{-sx} + \sqrt{-\frac{1}{2}\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right] - \frac{1}{2}\sqrt{\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right]^2 + r^2 e^{-2sx}\left(2b^2 + k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right)^2}} , \tag{8}$$

$$k_{2r} = b - \frac{re^{-sx}\left(2b^2 + k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right)}{2\sqrt{-\frac{1}{2}\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right] - \frac{1}{2}\sqrt{\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right]^2 + r^2 e^{-2sx}\left(2b^2 + k_0^2 e^{sx}\sum\limits_{i=0}^{n} A_i x^i\right)^2}}} , \tag{9}$$

$$k_{2i} = be^{-sx} - \sqrt{-\frac{1}{2}\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right] - \frac{1}{2}\sqrt{\left[b^2\left(1 - r^2 e^{-2sx}\right) - k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right]^2 + r^2 e^{-2sx}\left(2b^2 + k_0^2 e^{sx}\sum_{i=0}^{n} A_i x^i\right)^2}} , \tag{10}$$

$b = \dfrac{s}{2\left(1 + r^2 e^{-2sx}\right)}$, $\gamma_1(x) = k_1(x) + x\dfrac{\mathrm{d}k_1(x)}{\mathrm{d}x}$ and $\gamma_2(x) = k_2(x) + x\dfrac{\mathrm{d}k_2(x)}{\mathrm{d}x}$.

Standing wave ratio, VSWR, can be expressed as a function of the modulus of the voltage reflection coefficient R

$$\mathrm{VSWR} = \frac{1 + |R|}{1 - |R|}, \tag{11}$$

where

$$R = \frac{Z_p - Z_C}{Z_p + Z_C} . \tag{12}$$

3. NUMERICAL RESULTS

Real exponential line with copper conductors is considered. Resistance per unit length is calculated as $R' = \dfrac{1}{r_0\pi}\sqrt{\dfrac{\omega\mu}{2\sigma}}$. The value is R'=1.43837 Ω/m and $f = 300\mathrm{MHz}$. Wire radius is $r_0 = 1\mathrm{mm}$.

If the input impedance is transformed from $Z_C(0) = 100\Omega$ to output impedance of $Z_C(d) = 200\Omega$ then taper ratio is $P = 2$. Taper ratio is $P = 3$ for $Z_C(d) = 300\Omega$; $P = 4$ for $Z_C(d) = 400\Omega$ and $P = 5$ for $Z_C(d) = 500\Omega$. These values are usually used in practice.

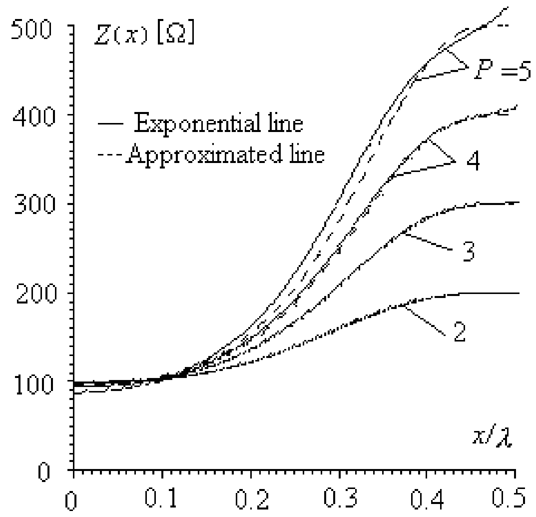

Fig. 2 Input impedance versus x/λ at 100 MHz

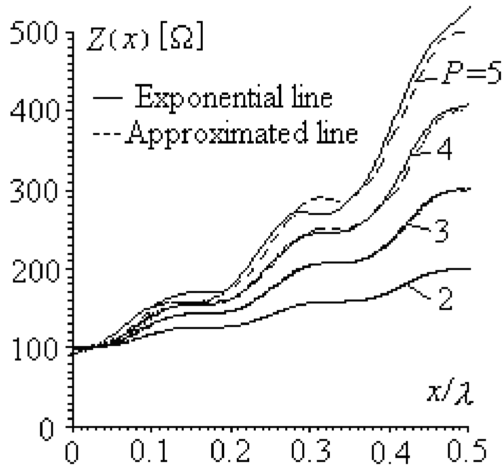

Fig. 3 Input impedance versus x/λ at 300 MHz

The satisfactory accuracy is obtained for the approximation of *C’* by the polynomial of fifth order (*n*=5), for wide frequency range and different taper ratios. Results for λ/2-transmission line are presented in Figs. 2-4. They show comparison between input impedances of exponential and approximated exponential transmission line, as the function of axial coordinate, for different taper ratio. For lower taper ratios polynomial degree can be lower, and for greater taper ratios greater degree should be chosen in order to obtain better approximation. At very high frequencies (Fig. 4) input impedance along exponential line has almost ideal exponential form. In that case there is very good agreement for all taper ratios. Fig. 5 presents standing wave ratio, VSWR , at the input end versus frequency, for different taper ratio for the exponential line.

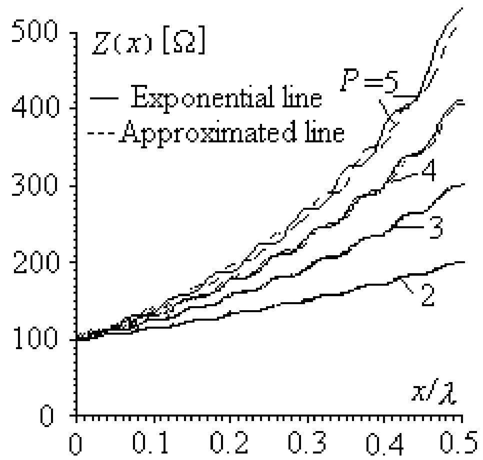

Fig. 4 Input impedance versus x/λ at 900 MHz

4. CONCLUSION

The loss exponential transmission line with capacitance per unit length approximated by power series is observed in this paper. Detailed analysis shows that input impedance depend on power of the series, frequency and taper ratio. It can be concluded that for lower taper ratios there is a good agreement in the large frequency range. The degree of polynomial approximation of the capacitance per unit length can be lower for lower taper ratios. For greater taper ratios there is better agreement at higher frequencies. For greater taper ratios and lower frequencies the deviation can be decreased using higher degree of polynomial approximation of the capacitance per unit length. The exponential transmission line acts as an excellent impedance transformer for a large frequency range, except at very low frequencies.

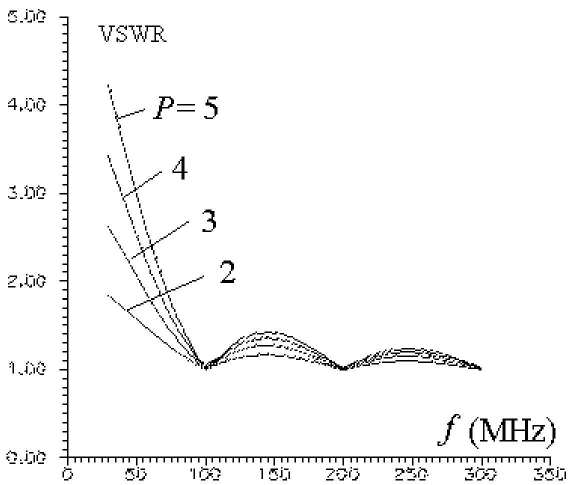

Fig. 5 VSWR versus frequency for different taper ratio

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BIOGRAPHIES

Zlata Cvetkovic was born in Blace, Serbia. She received the M.Sc. (1991) and Ph.D. (2002) degrees from the University of Nis. From 2002 she has been working as an assistant professor at the Department of Theoretical Electrical Engineering at the Faculty of Electronic Engineering in Nis. She is author or co-author of more than 60 papers and co-author of several textbooks. Her scientific research is focused on the analysis of the nonuniform transmission lines, modeling, analysis and design of electrostatic systems for generating uniform fields and for electrostatic space protection and numerical methods in applied electromagnetics.

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