A BRIEF INTRODUCTION TO CONTROL DESIGN DEMONSTRATED ON LABORATORY MODEL SERVO DR300 – AMIRA

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SUMMARY

This paper is written as a motivation for the students who are beginning to study the control engineering. Therefore, the below mentioned references are introduced with regard to this fact. The paper shortly presents the procedure of the control design, including a description of a system, an identification of its parameters, a simple and an advanced controller design. The controller design procedures are illustrated on the laboratory model servomechanism DR300 – AMIRA, which is held in the laboratory of control theory K26, Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University in Prague ⟨http://www.cvut.cz/⟩.

Keywords: *Systems, PID Controllers, Estimation, Kalman filters, LQG control*

1. INDRODUCTION

There are many different processes in practise and we usually want to control their properties, for instance, we want the rotation speed of a motor to be equal to some value for any load torque, we want planes not to fall down, we want to increase the electric power of power plants while decreasing the air pollution, we want to increase the capacity of hard discs so we have to control the reading machinery more accurately, etc.

The regulation is based on feedback control, see Fig. 1. The output of a system $y(t)$ is measured by a sensor. The controller computes the input of the system $u(t)$ based on the measured output $y(t)$ and its reference $w(t)$ and applies this value by an actuator to the system such that the output behaves how we want i.e. how we set the reference. For example, the brain observes our position by eyes and stimulates our muscles so we do not fall down.

For a control design, we have to know how the system behaves; we do the identification [13] and describe the system by mathematical tools usually. Very often, we do a physical setup and get a system of differential (recurrent) and algebraic equations at first. The second step is to determine the system inputs and outputs, to derive a so called *model* from the mathematical equations of the system and, if needed, to linearize it [6]. Then we can measure the system parameters and write down the complete model with numerical values.

There are two main approaches to linear systems in the control theory: state-space [9] and polynomial methods [12]. The former uses four matrices and is often written as a system of differential (recurrent) and algebraic equations. The latter expresses a linear model as a transfer function. *Single Input Single Output* (SISO) system is described by a fraction of two polynomials, i.e. ratio of Laplace transform of the output and Laplace transform of the input with zero initial conditions in continuous-time case and ratio of Z-transform of the output and Z-transform of the input with zero initial conditions in discrete-time case. *Multi Input Multi Output* (MIMO) system is described by a fraction of two polynomial matrices [9].

When we have the model of our system, we can design a controller. There are many ways how to do it. The controller design depends on what we want to achieve, for example, stability of the system and quality of the system behaviour, optimal behaviour of the system according to a criterion, etc.

This paper presents identification of the servomechanism DR300 – AMIRA (see 2. Fig. 2), which is held in laboratory K26, Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University (CTU) in Prague. Then several controllers are designed and applied to the system, for example, PID controller [2], LQ controller [1] with Kalman filter [10], [11].

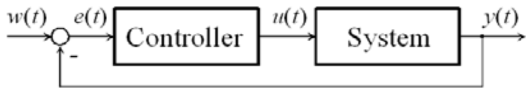

Fig. 1 The feedback control loop

The outline of the paper is as follows: in Section 2, the servomechanism DR300 – AMIRA is described. In Section 3, the model identification is performed. Section 4 presents the application of several controllers for the servomechanism DR300 – AMIRA.

2. SERVO DR300 – AMIRA

Servo DR300 – AMIRA (see Fig. 2) is a servomechanism consisting of two identical motors, which are connected by a mechanical clutch. The first motor is used for control of the rotation speed or the shaft angle. The second one, in the following called as a generator, is used for a simulation of load torque.

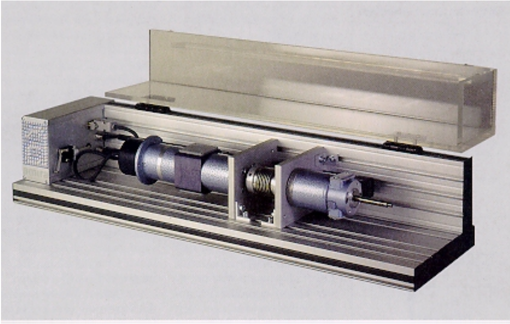

Fig. 2 The servomechanism DR300 – AMIRA

The whole system consists of three components. The first part is an I/O card MF614 (Humusoft ⟨http://www.humusoft.cz/⟩) with analogue and digital inputs and outputs. The second, power part, contains sources, current sensors and amplifiers. The third part contains two motors and sensors of the rotation speed and the shaft angle.

The system is controlled by the Real Time Toolbox of MATLAB ⟨http://www.mathworks.com/⟩.

Note that the servomechanism is relatively small and its behaviour is nonlinear for low values of input voltage; its mathematical description by the linear control theory is very inaccurate. Therefore, we control the rotation speed in the linear area only.

3. IDENTIFICATION OF THE SYSTEM

As it has been said, we have to know how our system behaves for a control design; we have to have a model of the system. This part of the control theory is taught at the CTU in Prague in the subject Systems and Model [6].

For the control, we consider one input (voltage of the motor) and one output (rotation speed). In this section, we introduce differential equations which can describe behaviour of a direct current (DC) motor. Then we identify a transfer function from the input voltage to the output rotation speed of the motor.

3.1. Mathematical Description of a DC Motor

A direct current motor with a permanent magnet can be described by differential equations

$$\begin{aligned} \frac{d\,i(t)}{dt} &= -\frac{R}{L}i(t) - \frac{k_e}{L}\omega(t) + \frac{1}{L}u(t) \\ \frac{d\omega(t)}{dt} &= -\frac{k_m}{J}i(t) - \frac{b}{J}\omega(t) - \frac{1}{J}m_z(t), \end{aligned} \qquad (1)$$

where $i(t)$ [A], $\omega(t)$ [rad s^{-1}], $u(t)$ [V], m_z[N m] are the current of the motor, rotation speed, input voltage and load torque respectively, and R, L, k_e, k_m, b, J are motor parameters. Equations (1) are called, in the control theory, state space model [9], [6].

As it has been said above, we consider that the voltage $u(t)$ is the input (the manipulated variable) of the system and the rotation speed $\omega(t)$ is the output (controlled variable) of the system. Instead of continuous-time state space description (1), we can use a transfer function in Laplace variable s (complex variable) [9], [6].

The transfer function is expressed as a fraction of Laplace transform of the output rotation speed $\Omega(s)$, and Laplace transform of the input voltage $U(s)$

$$G(s) = \frac{\Omega(s)}{U(s)} = \frac{\frac{k_m}{LJ}}{s^2 + \left(\frac{R}{L} + \frac{b}{J}\right)s + \frac{Rb + k_e k_m}{LJ}}. \qquad (2)$$

3.2. Identification of the System DR300 – AMIRA

For the identification of the system, we perform some experiments. At first, we generate a suitable input signal, see Fig. 3, apply it to the system and measure the output rotation speed, see Fig. 4. Note that the input signal provides operation in a linear area and excites the system inside a sufficient frequency band.

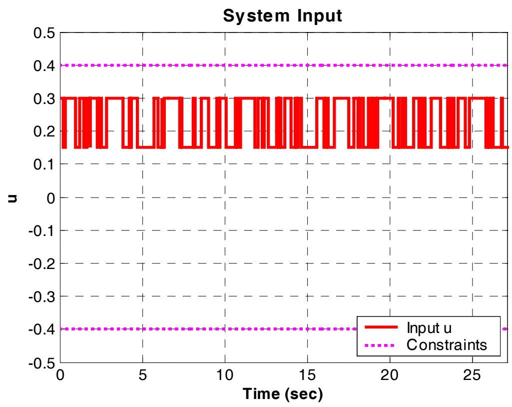

Fig. 3 The input signal for system identification

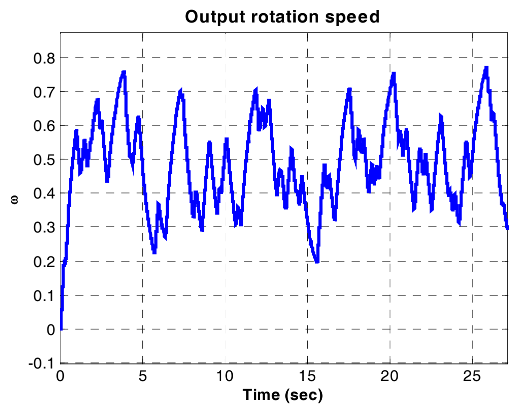

Fig. 4 The output response to the input identification signal

Using the least squares method [14], [6] we can compute the transfer function coefficients from the input and output sequences in Fig. 3 and Fig. 4. The resulting continuous transfer function is

$$G(s) = \frac{247.5}{(s+33.98)(s+1.402)}. \tag{3}$$

Now we must validate if model (3) is correct and sufficiently accurate. By comparing the step responses of the system and its model (3), see Fig. 5, we can see that our model is very accurate.

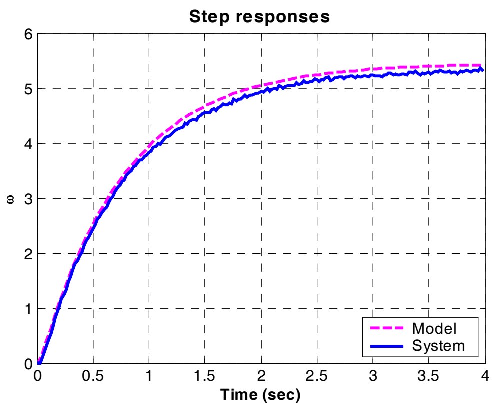

Fig. 5 The step responses – system and its model

Note that the step response of the system is measured in the linear area; the motor is spun and when the rotation speed reaches some steady-state value, the step change of the input voltage was applied to its input.

There are many identification methods. The method that we chose depends mainly on the characteristics of the system. For example, it is not possible to identify a nuclear reactor by using the input sequence from Fig. 3.

4. CONTROLLER DESIGN FOR THE SYSTEM

As it has been said, we can design many controllers for a system. For the design, we can use either the state space description or the transfer function. We can design either a continuous controller consisting of integrators, amplifiers and summators or a discrete controller, which is usually represented by a digital computer.

In this section, we shortly describe the main principles of several controllers (PID controllers, LQ controllers, Kalman filter for estimation of the system states) and apply these controllers to the servomechanism DR300 – AMIRA. The comparison of these regulations is shown. As it has been said above, we consider one manipulated variable (input voltage u) and one controlled variable (output rotation speed ω).

4.1. PID Controller

The PID block is a continuous controller consisting of three blocks: proportional, integral and derivative [2], [8]. The design of the PID controller is based on the continuous transfer function of the system. For the design, we can use for example frequency methods, the root locus method [8], etc. The theory of the PID controllers is taught at the CTU in Prague in the subject Systems and Control [8], [3].

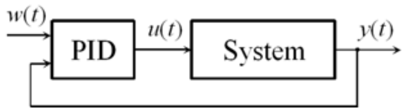

Fig. 6 The closed loop with the PID controller

The closed loop behaviour with the PID controller, which is designed by using the root locus method, is shown in Fig. 7. Note that in approximately 6 seconds, the step change of the generator input was applied to the servomechanism which simulates the change of the load torque.

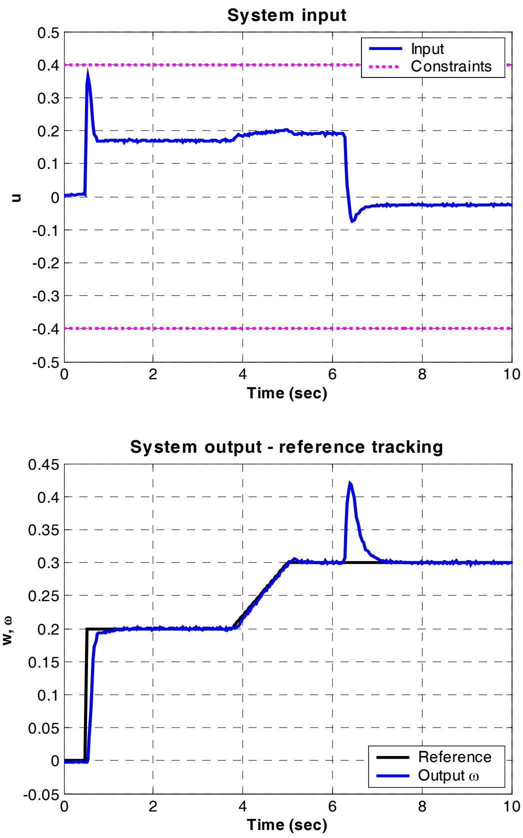

Fig. 7 The input (a) and output (b) responses – closed loop behaviour with the PID controller

4.2. LQ Controller with Kalman Filter

The main idea of an LQ controller design is the minimization of a quadratic criterion that is weighting the square of a manipulated variable and the square of a controlled variable [1]. Note that LQ means a linear system and a quadratic criterion.

In this paper, we use the discrete LQ controller design based on the state space description. The inputs of this controller are system states, but we measure only the output rotation speed. So we use the Kalman filter [9] for estimating these states.

The Kalman filter is the optimal state observer [9], [16], whose design is based on covariance of noise [10]. The inputs of the Kalman filter are system inputs and the system outputs, and the outputs of the Kalman filter are estimations of the system states.

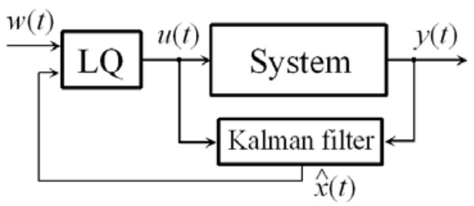

Fig. 8 The closed loop with the LQ controller

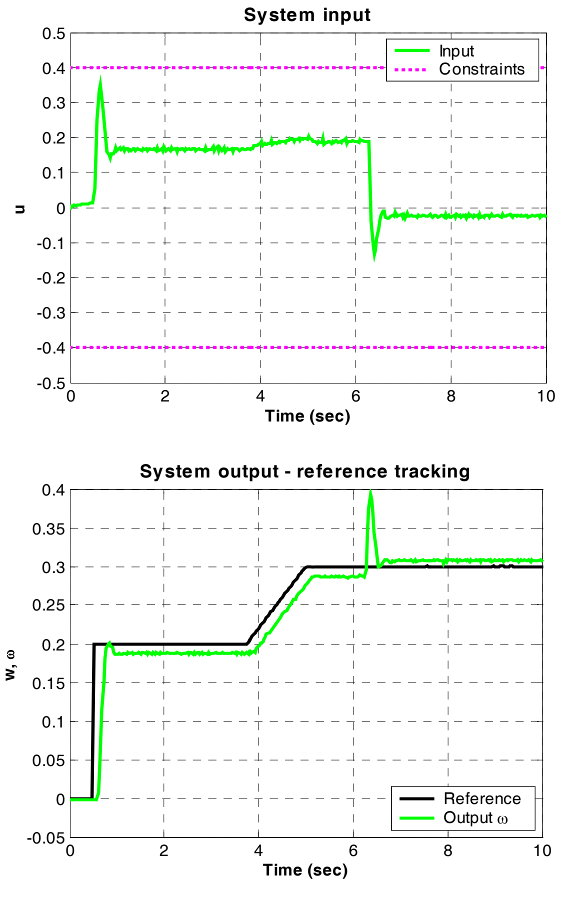

Fig. 9 The input (a) and output (b) responses – closed loop behaviour with the LQ controller and the Kalman filter

The closed loop behaviour with the LQ controller and the Kalman filter are shown in Fig. 9. If we observe Fig. 9b carefully, we discover that, unlike in PID control (4.1. Fig. 7b), the controlled variable (output rotation speed) does not asymptotically track the reference signal, because the steady state errors of the estimates of the system states are not equal to zero. This problem is solved in the next subsection.

The theory of dynamical systems and analysis of the dynamical systems are taught at the CTU in Prague in the subject Theory of Dynamical Systems [16], [7]. The theory of the LQ controller design and the Kalman filter design are taught at the CTU in Prague in the subjects Modern Control Theory [4], [15] and Estimation and Filtering [5].

4.3. LQ Controller with Kalman Filter with the Load Torque Estimation

As it has been said above, the classical Kalman filter is not able to reach zero errors of the estimates of the system states. The reason of this is that the Kalman filter inputs are only the system input and the system output. The Kalman filter does not know about the load torque which is caused by friction or by the generator (the second motor of the servomechanism DR300 – AMIRA).

Therefore, the Kalman filter design is modified such that the Kalman filter not only estimates system states, but the load torque, too. From equation (2), we can write

$$\ddot{\omega}(t) = -\left(\frac{R}{L}+\frac{b}{J}\right)\dot{\omega}(t) - \frac{Rb+k_e k_m}{LJ}\omega(t) + \frac{k_m}{LJ}u(t). \tag{4}$$

From equations (1) follows that the derivative of the rotation speed is proportional to the load torque m_z

$$\dot{\omega}(t) \approx -\frac{1}{J}m_z(t). \tag{5}$$

We modify model (4) by equation (5) (see Fig. 10) and use this modified model for the Kalman filter design [10]. Note that we suppose the load torque to be a random walk [5].

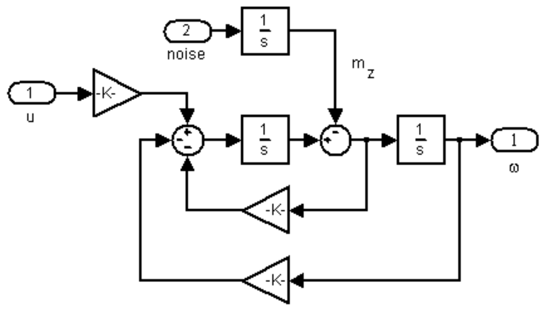

Fig. 10 Modified model for estimation of the unmeasured load torque

The closed loop behaviour with the LQ controller and modified Kalman filter are shown in Fig. 11. In Fig. 11b, you can see that in this case, the controlled variable asymptotically tracks the reference signal, because the modified Kalman filter estimates the system states and the load torque too, see Fig. 12.

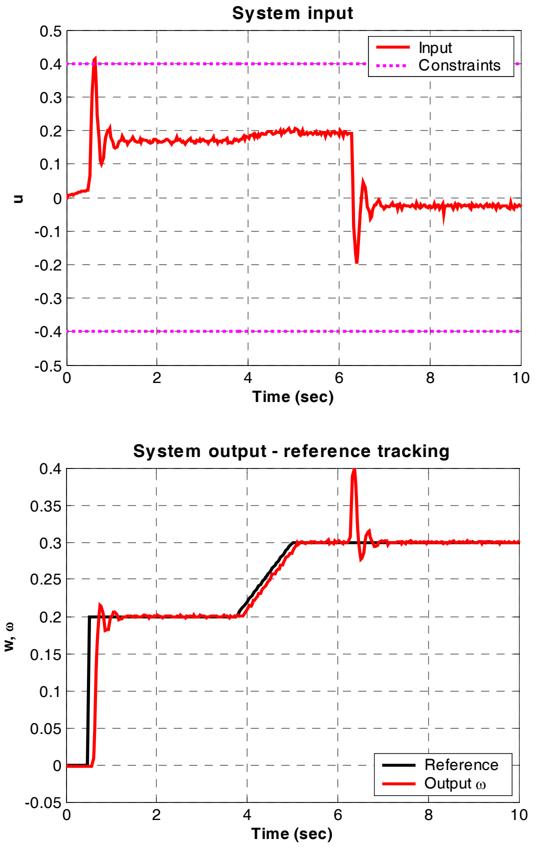

Fig. 11 The input (a) and output (b) responses – closed loop behaviour with the LQ controller and the Kalman filter + the estimation of the load torque

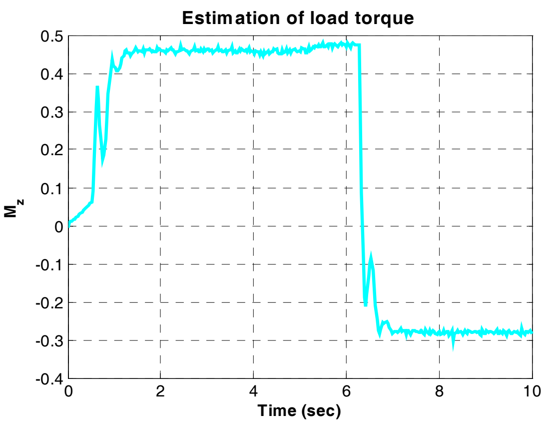

Fig. 12 The estimation of the load torque

5. CONCLUSION

This paper has been written for students as a motivation for studying the control engineering. The control design for the laboratory model servomechanism DR300 – AMIRA, including the description of the system, identification of its parameters, a simple and an advanced controller design, is presented.

For the system identification, the least squares method is used and the transfer function between the input voltage $u(t)$ and the output rotation speed $\omega(t)$ is obtained. The obtained model is used for the PID controller design and the LQ controller design with the Kalman filter for the system states estimation.

The classical Kalman filter is not able to reach zero steady state of the output estimation error, the system output does not track our reference signal. Therefore, the Kalman filter is modified for estimating of the load torque of the servomechanism. The comparison of the particular closed loops is shown in Fig. 13.

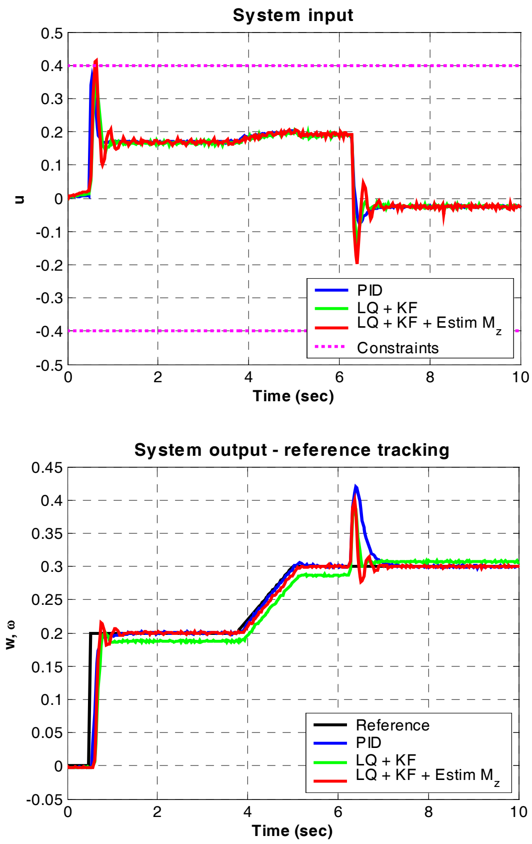

Fig. 13 The input (a) and output (b) responses – comparison of the controllers

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BIOGRAPHY

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