THE ALGORITHM FOR CALCULATION OF THE DYNAMIC RELIABILITY INDICES IN MULTI-STATE SYSTEMS

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SUMMARY

A lot of complex sytems (Operation Systems, Database systems, Distributed Systems, Information Systems, etc.) is possible describe, form the point of view of reliability, like the Multi-State Systems. The Multi-State System is investigated in this paper as an object of reliability analysis. In this case system and its component may experience more than two states of reliability as opposed to the Binary System (the system and its components are allowed to have only two possible states: completed failure and perfect functioning). The mathematical model of the Multi-State System is improved. We examine the system to allow a different number of discrete states for the system and for each component and propose to apply Dynamic Reliability Indices for investigation of this system. These indices estimate influence upon the Multi-State System reliability a change of the system component state.

Keywords: *System, distributed systems, multi-sate system, dynamic reliability*

1. INTRODUCTION

There are same mathematical models in the reliability analysis. Firstly, it is the Binary System: the system and its components are allowed to have only two possible states (completed failure and perfect functioning). Secondly, it is the *Multi-State System* (MSS). In a MSS, both the system and its components may experience more than two states, for example, completely failed, partially functioning and perfect functioning. The reliability analysis of Binary System has served as a foundation for the mathematical treatment of reliability theory. Many problem of the Binary System have been settled. But this approach fails to describe many situations where the system can have more than two distinct states [1, 2]. A MSS reliability analysis is a more flexible approach to evaluate system reliability.

In paper [6] the new class of reliability indices was determined and was named *Dynamic Reliability Indices* (DRI). Two groups of DRI were obtained. There are the group of deterministic and the group of probabilistic indices. The deterministic indices define the sets of the boundary states of MSS. The other group of indices reveals the probability of system failure or it's repairing. Another group of DRI was proposed in paper [7]. Indices of this group examine the influence of modifications of every system component to the system reliability.

However it is necessary to note, the MSS was investigated in [5 – 7] when levels of the component state are equals. This lack removes in this paper and DRI are defined for MSS to allow a different number of discrete states for the system and for each component.

The general model of the MSS is considered in this paper, it is *k*-out-of-*n* system. The *k*-out-of-*n* MSS with *n* components works if at least *k* components work. Both series and parallel systems are special case of *k*-out-of-*n* system: a series system is *n*-out-of-*n* system and a parallel system is 1-out-of-*n* system. Some computational access is described in [11-13].

2. MATHEMATICAL MODEL OF MSS

MSS it is system which consist of *n* components that denote as x_i (i = 1, …, *n*). Each component in the component set can take on either of states: from 0 (it is the complete failure) to m_i-1 (it is the perfect functioning). The dependence of the system reliability (system state) on its components state is defined by the structure function identically [2, 8, 9]:

$$\phi(x_1,\dots,x_2)=\phi(\boldsymbol{x})=\begin{cases}0 \text{ if } \boldsymbol{x} \text{ is a failed system state,}\\ s \text{ if } \boldsymbol{x} \text{ is an functioning system state.}\end{cases}$$

$s\in\{1, \dots, M\text{-}1\}$ and $x_i \in \{0, \dots, m_i\text{-}1\}$.

The structure function is determined as

$$\phi(\boldsymbol{x})\text{: } \{0, \dots, m_1\text{-}1\}\times\dots\times\{0, \dots, m_n\text{-}1\} \rightarrow \{0, \dots, M\text{-}1\}, \quad (1)$$

$m_i \neq m_j \neq M$ in generally.

As a rule the MSS in reliability analysis is a coherent system therefore the structure function (1) have assumptions [2, 4, 9]:

a) the structure function is nondecreasing and so: $\phi(s, \dots, s) = s$, $\phi(0, \dots, 0) = 0$ and $\phi(\boldsymbol{x}) \leq \phi(\boldsymbol{x}')$ if $\boldsymbol{x} \leq \boldsymbol{x}'$,;
b) the behavior of each component is mutually s-independent;
c) the every component relevant to the system.

In previous papers [2 – 4, 8, 9] the structure function is used for the estimation the probability of the different system state: $\Pr\{\phi(\boldsymbol{x})=l\}$, $l\in\{0, \dots, M\text{-}1\}$. The mathematical approach for investigation of dynamic behavior of MSS is evolved in this paper and the probability of change of system state $\mathrm{P}\{\phi(\boldsymbol{x}) = j \rightarrow \phi(\boldsymbol{x}) = l\}$ is calculated in this paper for estimation of system reliability. First this approach was suggested in [5 – 7]. It was founded on the

Logical Differential Calculus and the Direct Partial Logic Derivative in particular. But the Direct Partial Logic Derivative that was use in [5 – 7] is defined for function that variables values belong to the same set: $m_i = m_j = M$ and x_i, $\phi(\mathbf{x}) \in \{0, \ldots, M\text{-}1\}$. So, need to extend the Direct Partial Logic Derivative for function $\phi(\mathbf{x})$ where $x_i \in \{0, \ldots, m_i\text{-}1\}$ and $m_i \neq m_j \neq M$, in other words, for the system to allow a different number of discrete states for the system and for each component.

The Direct Partial Logic Derivative $\partial\phi(j\rightarrow k)/\partial x_i(a\rightarrow b)$ of function $\phi(\mathbf{x})$ (1) with respect to variable x_i reflects the fact of changing of function from j to l when the value of variable x_i is changing from a to b:

$$\partial\phi(j\rightarrow l)/\partial x_i(a\rightarrow b) = \\ = \phi(x_1,\ldots,x_{i\text{-}1}, a, x_{i+1},\ldots,x_n) \bullet \phi(x_1,\ldots,x_{i\text{-}1}, b, x_{i+1},\ldots,x_n) \quad (2)$$

where $j, l \in \{0, 1, \ldots, M\text{-}1\}$ and $a, b \in \{0, 1, \ldots, m_i\text{-}1\}$; $\bullet$ is the symbol of a comparison operation:

$$\frac{\partial\phi(j\rightarrow l)}{\partial x_i(a\rightarrow b)} = \begin{cases} m_i - 1, \text{if } \phi(x_1,\ldots,x_{i\text{-}1}, a, x_{i+1},\ldots,x_n) = j \ \& \\ \qquad \phi(x_1,\ldots,x_{i\text{-}1}, b, x_{i+1},\ldots,x_n) = l, \\ 0, \text{ in the other case.} \end{cases}$$

So, the Direct Partial Logic Derivative of the structure function allows to examine the influence of the i-th component state change into the system reliability. In other words this derivative discovers system states that are transformed as a result of the change of the component state. Followings Direct Partial Logic Derivatives are of interest for reliability analysis of MSS:

- $\partial\phi(1\rightarrow 0)/\partial x_i(a\rightarrow b)$ for $a \in \{1, \ldots, m_i\text{-}1\}$ and $b \in \{0, \ldots, m_i\text{-}2\}$, $b < a$;
- $\partial\phi(0\rightarrow l)/\partial x_i(c\rightarrow d)$ for $l, d \in \{1, \ldots, m_i\text{-}1\}$ and $c \in \{0, \ldots, m_i\text{-}2\}$, $c < d$.

The first derivatives is mathematical model of the system failure if i-th component state changes from a to b. Because the structure function $\phi(\mathbf{x})$ is nondecreasing this derivatives is $\partial\phi(1\rightarrow 0)/\partial x_i(a\rightarrow a\text{-}1)$ where $a\in\{1, \ldots, m_i\text{-}1\}$. Another assumptions for structure function (1) and experimental investigations in [10] conditioned on the next Direct Partial Logic Derivative for modelling of the system failure: $\partial\phi(1\rightarrow 0)/\partial x_i(1\rightarrow 0)$.

The second derivative permits the mathematical description the system renewal. There are two variants of investigation for the system repairing. Firstly it is the system repairing by the replacement of the failure component. This situation is determined by the Direct Partial Logic Derivative $\partial\phi(0\rightarrow l)/\partial x_i(0\rightarrow m_i\text{-}1)$. Secondly it is increase of component state, that is described in terms as $\partial\phi(0\rightarrow 1)/\partial x_i(c\rightarrow c\text{+}1)$. However, the first variant is the more important for application. Because the structure function of the MSS is nondecreasing this derivative can be to assign as $\partial\phi(0\rightarrow 1)/\partial x_i(0\rightarrow m_i\text{-}1)$.

So, for analysis of the MSS dynamic behavior is need to use Direct Partial Logic Derivative

- $\partial\phi(1\rightarrow 0)/\partial x_i(1\rightarrow 0)$;
- $\partial\phi(0\rightarrow 1)/\partial x_i(0\rightarrow m_i\text{-}1)$.

For example, show the MSS 2-out-of-3 where structure function $\phi(\mathbf{x})$ depends of three variables (n=3) and has $m_1 = 4$, $m_2 = 2$, $m_3 = 3$, $M = 3$ (Table 1). Compute Direct Partial Logic Derivatives $\partial\phi(1\rightarrow 0)/\partial x_i(1\rightarrow 0)$ (i = = 1, …, n) of this function $\phi(\mathbf{x})$ for analysis of the system failure.

$x_1\,x_2\,x_3$	$\phi(\mathbf{x})$	$x_1\,x_2\,x_3$	$\phi(\mathbf{x})$	$x_1\,x_2\,x_3$	$\phi(\mathbf{x})$	$x_1\,x_2\,x_3$	$\phi(\mathbf{x})$
0 0 0	**0**	1 0 0	**0**	2 0 0	**0**	3 0 0	**0**
0 0 1	**0**	1 0 1	**1**	2 0 1	**1**	3 0 1	**1**
0 0 2	**0**	1 0 2	**1**	2 0 2	**1**	3 0 2	**1**
0 1 0	**0**	1 1 0	**1**	2 1 0	**1**	3 1 0	**1**
0 1 1	**1**	1 1 1	**1**	2 1 1	**2**	3 1 1	**2**
0 1 2	**1**	1 1 2	**2**	2 1 2	**2**	3 1 2	**2**

Table 1 The example of the structure function of the MSS 2-out-of-3

According to (2) for this function it is necessary to analyze values of the function when x_i=0 and x_i=1:

- $\partial\phi(1\rightarrow 0)/\partial x_1(1\rightarrow 0) = \phi(1, x_2, x_3) \bullet \phi(0, x_2, x_3)$;
- $\partial\phi(1\rightarrow 0)/\partial x_2(1\rightarrow 0) = \phi(x_1, 1, x_3) \bullet \phi(x_1, 0, x_3)$;
- $\partial\phi(1\rightarrow 0)/\partial x_3(1\rightarrow 0) = \phi(x_1, x_2, 1) \bullet \phi(x_1, x_2, 0)$.

Therefore, the elements the Direct Partial Logic Derivative $\partial\mathbf{x}(1\rightarrow 0)/\partial x_1(1\rightarrow 0)$ are equal two if $\phi(\mathbf{x})$ =0 и $\phi(\mathbf{x})$ =1 for specified variables only (Table 2).

So, this MSS 2-out-of-3 is failure in states $x_1x_2x_3$ (Table 2):

a) 101, 102, 110 if the first component is breakdown;
b) 011, 012, 110, 210, 310 if the second component is failure;
c) 011, 101,201,301 if the third component is not functioning.

States for the system repairing is calculated by the similar method. For example, in Table 3 shown the Direct Partial Logic Derivative $\partial\phi(0\rightarrow 1)/\partial x_i(0\rightarrow m_i\text{-}1)$ and the analysis of this derivative permits to obtain states $x_1x_2x_3$ of the system failure for which the replacement of the broken component is restored the system:

a) 001, 002, 010 if the first component is replaced;
b) 001, 002, 100, 200, 300 if the second component is replaced;
c) 010, 100, 200, 300 if the third component is replaced.

Direct Partial Logic Derivatives allow to analyze dynamic properties MSS, which is submitted as structural function. The class of indices is proposed to use for reliability analysis of MSS below.

$x_1\,x_2\,x_3$	$\frac{\partial\varphi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\varphi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\varphi(1\to0)}{\partial x_3(1\to0)}$
0 0 0	**0**	**0**	**0**
0 0 1	**0**	**0**	**0**
0 0 2	**0**	**0**	**0**
0 1 0	**0**	**0**	**0**
0 1 1	**0**	**2**	**2**
0 1 2	**0**	**2**	**0**
1 0 0	**0**	**0**	**0**
1 0 1	**2**	**0**	**2**
1 0 2	**2**	**0**	**0**
1 1 0	**2**	**2**	**0**
1 1 1	**0**	**0**	**0**
1 1 2	**0**	**0**	**0**
2 0 0	**0**	**0**	**0**
2 0 1	**0**	**0**	**2**
2 0 2	**0**	**0**	**0**
2 1 0	**0**	**2**	**0**
2 1 1	**0**	**0**	**0**
2 1 2	**0**	**0**	**0**
3 0 0	**0**	**0**	**0**
3 0 1	**0**	**0**	**2**
3 0 2	**0**	**0**	**0**
3 1 0	**0**	**2**	**0**
3 1 1	**0**	**0**	**0**
3 1 2	**0**	**0**	**0**

Table 2 The example of the Direct Partial Logic Derivative $\partial\phi(1\to0)/\partial x_i(1\to0)$

$x_1\,x_2\,x_3$	$\frac{\partial\varphi(0\to1)}{\partial x_1(0\to3)}$	$\frac{\partial\varphi(0\to1)}{\partial x_2(0\to1)}$	$\frac{\partial\varphi(0\to1)}{\partial x_3(0\to2)}$
0 0 0	**0**	**0**	**0**
0 0 1	**2**	**2**	**0**
0 0 2	**2**	**2**	**0**
0 1 0	**2**	**0**	**2**
0 1 1	**0**	**0**	**0**
0 1 2	**0**	**0**	**0**
1 0 0	**0**	**2**	**2**
1 0 1	**0**	**0**	**0**
1 0 2	**0**	**0**	**0**
1 1 0	**0**	**0**	**0**
1 1 1	**0**	**0**	**0**
1 1 2	**0**	**0**	**0**
2 0 0	**0**	**2**	**2**
2 0 1	**0**	**0**	**0**
2 0 2	**0**	**0**	**0**
2 1 0	**0**	**0**	**0**
2 1 1	**0**	**0**	**0**
2 1 2	**0**	**0**	**0**
3 0 0	**0**	**2**	**2**
3 0 1	**0**	**0**	**0**
3 0 2	**0**	**0**	**0**
3 1 0	**0**	**0**	**0**
3 1 1	**0**	**0**	**0**
3 1 2	**0**	**0**	**0**

Table 3 The example of the Direct Partial Logic Derivative $\partial\phi(0\to1)/\partial x_i(0\to m_i)$

3. DYNAMIC DETERMINISTIC RELIABILITY INDICES

DDRI evaluate the influence a change of the component state on system reliability. They are defined as sets of the boundary states of the system. Here the boundary state of the system is the system state $s_1 \ldots s_i \ldots s_n$ when the modification of i-th component state from s_i into s'_i causes to the system failure or the system repairing.

Definition 1. DDRI are sets of the boundary state of the system $\{G_f\}$ (for system failure) and $\{G_r\}$ (for system repairing) [5, 10]:

$$\{G_f\} = \{G_f | x_1\} \cup \{G_f | x_2\} \cup \ldots \cup \{G_f | x_n\}, \qquad (3)$$

$$\{G_r\} = \{G_r | x_1\} \cup \{G_r | x_2\} \cup \ldots \cup \{G_r | x_n\}, \qquad (4)$$

where subsets are

$$\{G_f | x_i\} \Leftrightarrow \{G_f | \partial\phi(1\to0)/\partial x_i(1\to0) \neq 0\}, \qquad (5)$$

$$\{G_r | x_i\} \Leftrightarrow \{G_r | \partial\phi(0\to1)/\partial x_i(0\to m_i\text{-}1)\}. \qquad (6)$$

$\{G_f | x_i\}$ and $\{G_r | x_i\}$ are subsets of the boundary state of the system for everyone of system component x_i. Therefore, it is necessary to analyze everyone component state s_i and to check the fact of MSS failure or repairing after the modification of this states. The Direct Partial Logic Derivatives (2) allow to formalize this procedure.

CDRI (Component Dynamic Reliability Indicis) are a probability evaluation of influence i-th a system component on a possibility of a failure or repairing system. A point of view of system reliability the unstable components are determined by these indices. CDRI are calculated by DDRI.

Definition 2. CDRI are probabilities of MSS failure and repairing at a modification of a state of i-th system component [7, 10]:

$$P_f(i) = p(i)_{1\to0}^{1\to0} \cdot p_1(i), \qquad (7)$$

$$P_r(i) = p(i)_{0\to m_i-1}^{0\to1} \cdot p_0(i), \qquad (8)$$

$p(i)_{1\to0}^{1\to0}$ is the structural probability of i-th component state modification from 1 to 0 where the system fail; $p_1(i)$ is the probability of state 1 of i-th component; $p(i)_{0\to m_i-1}^{0\to1}$ is the structural probability of i-th component replace for system repairing; $\cdot p_0(i)$ is the probability of state 0 of i-th component failure.

$$p(i)_{1\to0}^{1\to0} = \frac{\rho(i)_{1\to0}^{1\to0}}{m_1 m_2 \ldots m_n}, \qquad (9)$$

$$p(i)_{0\to m_i-1}^{0\to1} = \frac{\rho(i)_{0\to m_i-1}^{0\to1}}{m_1 m_2 \ldots m_n}, \qquad (10)$$

$\rho(i)_{1\to0}^{1\to0}$ is number of system states when the breakdown of the i-th component forces the system failure and $\rho(i)_{0\to m_i-1}^{0\to1}$ is number of system states when system repairing bring about by to replace i-th component.

Note, numbers $\rho(i)_{1\to 0}^{1\to 0}$ and $\rho(i)_{0\to m_i-1}^{0\to 1}$ are obtained as numbers of values of Direct Partial Logic Derivative $\partial\phi(1\to 0)/\partial x_i(1\to 0)$ and $\partial\phi(0\to 1)/\partial x_i(0\to m_i\text{-}1)$ whit respect to i-th variable which are not equal 0. In other words numbers $\rho(i)_{1\to 0}^{1\to 0}$ and $\rho(i)_{0\to m_i-1}^{0\to 1}$ are cardinality of the set $\{G_f | x_i\}$ in (5) and the set $\{G_r | x_i\}$ in (6) accordingly.

DIRI are generalization of DDRI and are probability evaluation of a modification of MSS reliability at a change of the system components state. In particular, the probability of boundary of system states is estimated by these indices.

Definition 3. DIRI is probability of the system failure or repairing if one of system component fails or restores [6, 10]:

$$P_f = \sum_{i=1}^{n} P_f(i) \prod_{\substack{q=1 \\ q\neq i}}^{n} (1 - P_f(q)), \tag{11}$$

$$P_r = \sum_{i=1}^{n} P_r(i) \prod_{\substack{q=1 \\ q\neq i}}^{n} (1 - P_r(q)), \tag{12}$$

where $P_f(i)$ and $P_r(i)$ is determined in (7) and (8).

4. THE ALGORITHM FOR CALCULATION OF THE DYNAMIC RELIABILITY INDICES

DRI are calculated for the estimation of the dynamic reliability analysis of MSS. As mentioned above, DRI are calculated by the Direct Partial Logic Derivative of the structure function. It is used in the algorithms of DRI calculation.

The Algorithm of DRI calculation.

Step 1. DDRI $\{G_f\}$ and $\{G_r\}$ are calculated for the MSS.

Step 1.1. The derivatives $\partial\phi(1\to 0)/\partial x_i(1\to 0)$ are calculated by (2) (i = 1, …, n).

Step 1.2. The subsets $\{G_f | x_i\}$ are obtained in accordance with (5).

Step 1.3. The set of the boundary states of the system $\{G_f\}$ (3) is formed.

Step 1.4. The derivative $\partial\phi(0\to 1)/\partial x_i(0\to m_i\text{-}1)$ are calculated on account of (2) (i = 1, …, n).

Step 1.5. The subsets $\{G_r | x_i\}$ are obtained by (6).

Step 1.6. The set (4) $\{G_r\}$ of the boundary states of the system is formed.

Step 2. CDRI $P_f(i)$ and $P_r(i)$ are calculated.

Step 2.1. The numbers $\rho(i)_{1\to 0}^{1\to 0}$ and $\rho(i)_{0\to m_i-1}^{0\to 1}$ are obtained. They are conformed to numbers nonzero elements of the Direct Partial Logic Derivatives $\partial\phi(1\to 0)/\partial x_i(1\to 0)$ and $\partial\phi(0\to 1)/\partial x_i(0\to m_i\text{-}1)$, that are calculated in step 1.2 and step 1.4 accordantly.

Step 2.2. The structural probability $p(i)_{1\to 0}^{1\to 0}$ of i-th component state modification from 1 to 0 where the system fail and the structural probability $p(i)_{0\to m_i-1}^{0\to 1}$ of i-th component replace for system repairing are calculated according to (9) and (10).

Step 2.3. CDRI (probabilities of MSS failure or repairing at a modification of a state of i-th system component) are obtained by (7) – (8).

Step 3. DIRI for MSS estimation the probability of the system failure and the system repairing by (11) and (12).

5. EXAMPLES

DRI are calculated for the MSS 2-out-of-3 (Table 1). These indices are determined accordantly the **Algorithm** of DRI calculation.

Step 1. DDRI $\{G_f\}$ and $\{G_r\}$ are calculated for the MSS.

Step 1.1. The derivatives $\partial\phi(1\to 0)/\partial x_i(1\to 0)$ are calculated by (2) (i = 1, 2, 3) and are presented in Table 2.

Step 1.2. The subsets $\{G_f | x_i\}$ are obtained in accordance with (5) and are:

$\{G_f | x_1\}$ = {101, 102, 110},
$\{G_f | x_2\}$ = {011, 012, 110, 210, 310},
$\{G_f | x_3\}$ = {011, 101, 201, 301}.

Step 1.3. The set of the boundary states of the system $\{G_f\}$ (3) is formed:

$\{G_f\}$ = {0**11**, 0**1**2, **1**0**1**, **1**02, **11**0, 20**1**, 2**1**0, 30**1**, 3**1**0}.

Step 1.4. The derivative $\partial\phi(0\to 1)/\partial x_i(0\to m_i\text{-}1)$ is calculated on account of (2) for i = 1, 2, 3 (Table 3).

Step 1.5. The subsets $\{G_r | x_i\}$ are obtained by (6):

$\{G_r | x_1\}$ = {001, 002, 010},
$\{G_r | x_2\}$ = {001, 002, 100, 200, 300},
$\{G_r | x_3\}$ = {010, 100, 200, 300}.

Step 1.6. The set (4) $\{G_r\}$ of the boundary states of the system is formed:

$\{G_r\}$ = {**00**1, **00**2, **0**1**0**, 1**00**, 2**00**, 3**00**}.

The set $\{G_f\}$ and the set $\{G_r\}$ are important for the reliability analysis of dynamic behavior of MSS. But DDRI stipulate for problem for the MSS of large dimensionality. CDRI and DIRI are preferable for applications.

Step 2. CDRI $P_f(i)$ and $P_r(i)$ are obtained.

Step 2.1. The numbers $\rho(i)_{1\to 0}^{1\to 0}$ and $\rho(i)_{0\to m_i-1}^{0\to 1}$ are:

$\rho(1)_{1\to 0}^{1\to 0} = 3$, $\rho(2)_{1\to 0}^{1\to 0} = 5$, $\rho(3)_{1\to 0}^{1\to 0} = 4$;
$\rho(1)_{0\to 3}^{0\to 1} = 3$, $\rho(2)_{0\to 1}^{0\to 1} = 5$, $\rho(3)_{0\to 2}^{0\to 1} = 4$.

Step 2.2. The structural probability of i-th component are calculated according to (9) and (10):

$p(1)_{1\to 0}^{1\to 0} = 0.125$, $p(2)_{1\to 0}^{1\to 0} = 0.208$, $p(3)_{1\to 0}^{1\to 0} = 0.167$;

$p(1)^{0\to1}_{0\to3} = 0.125, p(2)^{0\to1}_{0\to1} = 0.208, p(3)^{0\to1}_{0\to2} = 0.167;$

Step 2.3. CDRI are obtained by (7) – (8):

$P_f(1) = 0.019,\quad P_f(2) = 0.144,\quad P_f(3) = 0.057;$

$P_r(1) = 0.052,\quad P_r(2) = 0.144,\quad P_r(3) = 0.083.$

The probability of the component state is in Table 4.

Component	State			
	0	1	2	3
x_1	0.20	0.15	0.23	0.42
x_2	0.31	0.69	—	—
x_3	0.16	0.34	0.50	—

Table 4 Component state probability

CDRI illustrate relates the system states to the component states when change of *i*-th component forces a change in the system state. Therefore the analyze of CDRI shows:

a) The system has maximum probability of failure when the second component is failure because its CDRI has largest value $P_f(2)$ = 0.144;
b) The system fails with minimum probability if the first component have failed ($P_f(1)$ = 0.019);
c) MSS repairs with maximum probability by replacement of the second component as since CDRI $P_r(2)$= 0.143.

Step 3. DIRI for MSS estimation the probability of the system failure and the system repairing by (11) and (12).

DIRI permit to obtainer the probability of the system failure if one of the system components is breakdown. It is P_f = 0.196. The probability of the system repairing is P_r = =0.234 if one of the failure component of the system is replaced.

DRI are calculated for series (3-out-of-3) and parallel (1-out-of-3) systems. Basic data (the number of components, state levels of the component etc.) is similar as for MSS 2-out-of-3 that is investigated above.

The series system. DDRI are calculated by virtue of Direct Partial Logic Derivatives $\partial\phi(1\to0)/\partial x_i(1\to0)$ and $\partial\phi(0\to1)/\partial x_i(0\to m_i\text{-}1)$ as in previous time for MSS 2-out-of-3. The structure function for the series system is in Table 5.

$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$
0 0 0	**0**	1 0 0	**0**	2 0 0	**0**	3 0 0	**0**
0 0 1	**0**	1 0 1	**0**	2 0 1	**0**	3 0 1	**0**
0 0 2	**0**	1 0 2	**0**	2 0 2	**0**	3 0 2	**0**
0 1 0	**0**	1 1 0	**0**	2 1 0	**0**	3 1 0	**0**
0 1 1	**0**	1 1 1	**1**	2 1 1	**1**	3 1 1	**1**
0 1 2	**0**	1 1 2	**1**	2 1 2	**1**	3 1 2	**1**

Table 5 The structure function of the series system (MSS 3-out-of-3)

DDRI for series system are:

{Gf} = {**111**, **112**, **211**, 212, **311**, 312},

{Gr} = {**0**11, **0**12, 1**0**1, 1**0**2, 11**0**, 2**0**1, 2**0**2, 21**0**, 3**0**1, 3**0**2, 31**0**}.

DDRI are not obvious. So the CDRI and DIRI are used in practice. The calculation of CDRI for series system is presented in Table 6. So, the breakdown of the second component causes the maximum probability of the system failure ($P_f(2)$ = 0.173). The first component has an influence on the system failure least of all ($P_f(1)$ = 0.013). The system repairing is probably the most by the replacement of the second component ($P_r(2)$ = 0.173).

	Step 2.1		*Step 2.2*		*Step 2.3*	
	$\rho(i)^{1\to0}_{1\to0}$	$\rho(i)^{0\to1}_{0\to m_i}$	$p(i)^{1\to0}_{1\to0}$	$p(i)^{0\to1}_{0\to m_i}$	$P_f(i)$	$P_r(i)$
x_1	2	2	0.083	0.083	0.013	0.035
x_2	6	6	0.25	0.25	0.173	0.173
x_3	3	3	0.125	0.125	0.043	0.063

Table 6 CDRI calculation for series system

DIRI for series system are calculated in accordance with (15) and (16): P_f = 0.208 and P_r = 0.233.

The parallel system. The structure function of this system is in Table 7. The DDRI are calculated by the Direct Partial Logic Derivatives $\partial\phi(1\to0)/\partial x_i(1\to0)$ and $\partial\phi(0\to1)/\partial x_i(0\to m_i\text{-}1)$. They are: {Gf} = {00**1**, 0**1**0, **1**00} and {Gr} = {**000**}.

$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$	$x_1\,x_2\,x_3$	$\phi(\boldsymbol{x})$
0 0 0	**0**	1 0 0	**1**	2 0 0	**1**	3 0 0	**1**
0 0 1	**1**	1 0 1	**2**	2 0 1	**2**	3 0 1	**2**
0 0 2	**1**	1 0 2	**2**	2 0 2	**2**	3 0 2	**2**
0 1 0	**1**	1 1 0	**2**	2 1 0	**2**	3 1 0	**2**
0 1 1	**2**	1 1 1	**2**	2 1 1	**2**	3 1 1	**2**
0 1 2	**2**	1 1 2	**2**	2 1 2	**2**	3 1 2	**2**

Table 7 The structure function of the parallel system (MSS 1-out-of-3)

The calculation of CDRI for parallel system is presented in Table 8. The breakdown of the second component causes the maximum probability of the system failure ($P_f(2)$ = 0.029) as for previous system. The system repairing is probably the most by the replacement of the second component ($P_r(2)$ = 0.029) too.

	Step 2.1		*Step 2.2*		*Step 2.3*	
	$\rho(i)^{1\to0}_{1\to0}$	$\rho(i)^{0\to1}_{0\to m_i}$	$p(i)^{1\to0}_{1\to0}$	$p(i)^{0\to1}_{0\to m_i}$	$P_f(i)$	$P_r(i)$
x_1	1	1	0.042	0.042	0.006	0.018
x_2	1	1	0.042	0.042	0.029	0.029
x_3	1	1	0.042	0.042	0.014	0.021

Table 8 CDRI calculation for parallel system (MSS 1-out-of-3)

DIRI are probabilities of the change of the system reliability if the state of one of the system components is changed. The probability of the system failure if one of the component breakdown is $P_f = 0.049$ in accordance with (11). The probability of the system repairing obtained by (12) and is $P_f = 0.064$ if one of the failure component of the system is replaced.

These examples for series, parallel and 2-out-of-3 systems reveal the main point of dynamic indices CDRI and DIRI well. CDRI reflect the influence of the change of the specifically *i*-th component state to the system reliability. In particular the system failure and system repairing depending on the *i*-th component state modification are examined in this paper. Because the component state probabilities are equal the change of the system components have the similar influence to the system reliability in these examples. So the second component has the largest probability of the system failure if this component breaks down. The first component has the least probability in analogous situation for series, parallel and 2-out-of-3 systems. And the second component has the largest probability for the system repairing if the replacement its component.

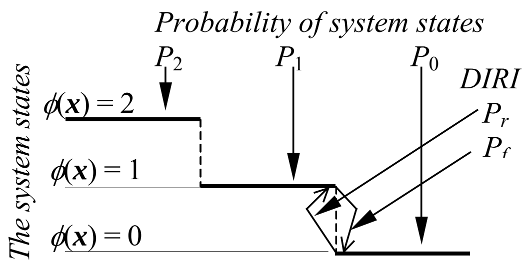

Fig. 1 DIRI and probability of system states

DIRI describe the dynamic characteristic of the MSS and are different for series, parallel and 2-out-of-3 systems (Table 9). The probability of the failure of the series system is highest possible ($P_f = 0.208$) if the one of the system component breaks down. The probability of the MSS failure P_f is minimum for parallel system. But this probability has the another meaning that the probability $P_l = \Pr\{\phi(\mathbf{x}) = l\}$, $l \in \{0, 1, \ldots, m\text{-}1\}$ (Fig. 1). It is clearly for the system repairing (the probability P_r in Table 9).

Indices		The series system	The system 2-out-of 3	The parallel system
Static	P_0	0.750	0.292	0.042
	P_1	0.250	0.500	0.250
	P_2	0	0.208	0.708
Dynamic (DIRI)	P_f	0.208	0.196	0.049
	P_r	0.233	0.234	0.064

Table 9 Reliability indices for series, 2-out-of-3 systems and parallel systems

There are 11 boundary states for repairing of the series system by the replacement of one of the failure system component. It is determined on account of $\{G_r\}$ for the series system. It is one state for parallel system only $\{G_r\} = \{\mathbf{000}\}$. The failure parallel system is reestablished by three possible ways: replacement of the first component, of the second component or the third component.

6. CONCLUSIONS

In this paper we presented new measure for MSS reliability, which is calculated by structure function of MSS model of network. This model allows to determine the some level of system availability (are not two only) in contrast to the Binary System. The MSS model is improved moreover: system has a different number of discrete states for the system and for each component.

This measure, which is named DRI, involves probabilities of the changes of the system states that are assigned by changes of component states. We shown two system changes: the system failure and system repair. But the suggesting method for reliability analysis of the MSS can be used to estimate the other changes in the system state.

In next investigations we are planning: to generalize this method for multi-input and multi-output network.

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