## SENSITIVITY ANALYSIS OF STATE VARIABLE ESTIMATORS FOR TWO-MASS DRIVE SYSTEM

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#### SUMMARY

The paper deals with the comparison of two estimation methods for torsional torque, load speed and disturbance torque reconstruction based on the Luenberger observer and Kalman filter for two-mass drive system. Estimation errors caused by the drive parameter changes were checked and compared. Due to a lack of analytical methods for gain matrices design for such estimators, parameters of both estimators were optimised using the same genetic-gradient algorithm and the same optimisation index. Such procedure has enabled the detailed comparison of results obtained for these two state estimators, based on simulation and experimental tests.

Keywords: Estimation techniques, control, DC drive, elastic joint, genetic algorithms

### 1. INTRODUCTION

In some industrial applications like the rolling mill drives, the mechanical part of the system has very low resonant frequency, because of a long shaft between the motor and the load machine. So, specially in the drive systems with high performances speed and torque regulation, the motor speed is different from the load speed during transients. The speed difference results in the coupling shaft stresses, which influences this mechanical coupling in a negative way. Additionally, speed oscillations cause decrease in the quality of the rolling material and can influence the stability of the control system [1-4]. The simplest method to eliminate the oscillation problem (occurring while the reference speed changes) is a slow change of reference velocity. But it causes the decrease of the drive system dynamics and does not against oscillations resulting protect while disturbance torque changes. Some methods of solving this problem are reported in technical papers. The most advanced techniques, ensuring very good performances of the system, are based on special control structures with additional feedbacks from such state variables as torsional torque, load speed and/or disturbance torque. But the direct feedbacks from these signals are very often impossible, because additional measurements of these mechanical variables are difficult, cost effective and reduce the system reliability. Thus special systems for estimation of these variables are necessary, as state observers or state filters. The tuning procedure of different estimator parameters is not described in detail; in some applications, like Kalman filters, this procedure is based on trial and error method and it is really hard to compare various methods without the performance index [5]-[7].

The main goal of this paper is the comparison of two estimation methods for torsional torque, load speed and disturbance torque reconstruction based on the Luenberger observer and Kalman filter, for nominal as well as for changed drive system parameters. Due to a lack of analytical methods for gain matrices design for such estimators, especially for the Kalman filter, parameters of both estimators were optimised using the same genetic-gradient algorithm and the same optimisation index. Such procedure has enabled the detailed comparison of the results obtained with these two state estimators, based on simulation and experimental tests.

# 2. THE MATHEMATICAL MODEL OF THE DRIVE SYSTEM

In the paper a commonly-used model of the drive system with the resilient coupling is considered. The system is described by the following state equations (in per unit system), with nonlinear friction neglected:

$$\frac{d}{dt} \begin{bmatrix} \omega_{1}(t) \\ \omega_{2}(t) \\ m_{s}(t) \end{bmatrix} = \begin{bmatrix} \frac{-d}{T_{m1}} & \frac{d}{T_{m1}} & \frac{-1}{T_{m1}} \\ \frac{d}{T_{m2}} & \frac{-d}{T_{m2}} & \frac{1}{T_{m2}} \\ \frac{1}{T_{c}} & \frac{-1}{T_{c}} & 0 \end{bmatrix} \begin{bmatrix} \omega_{1}(t) \\ \omega_{2}(t) \\ m_{s}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{T_{m1}} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} m_{e} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{T_{m2}} \\ 0 \end{bmatrix} \begin{bmatrix} m_{o} \end{bmatrix}$$
(1)

where:  $\omega_{1-}$  motor speed,  $\omega_{2}$  - load speed,  $m_{e^{-}}$  motor torque,  $m_{s^{-}}$  shaft (torsional) torque,  $m_{o^{-}}$  disturbance torque,  $T_{m1}$  – mechanical time constant of the motor,  $T_{m2}$  – mechanical time constant of the load machine,  $T_{c}$  – stiffness time constant, d – damping coefficient of the mechanical coupling.

Parameters of the analysed system are following:  $T_{m1}=230ms$ ,  $T_{m2}=230ms$ ,  $T_c=2.4ms$ , d=0,25.

The electromagnetic torque of the motor is used as a control input of the system and the angular speed of the motor is taken as the output value. Hence, results of this work can be applied to any kind of the electrical motor with high performance electromagnetic torque control.

# 3. METHODS OF MECHANICAL STATE VARIABLES ESTIMATION

### 3.1 Mathematical model of the extended fullorder Luenberger observer for two-mass drive system

For the linear dynamical system described by linear state equation:

$$\frac{d}{dt}\underline{x}(t) = \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t)$$

$$\underline{y}(t) = \underline{C}\underline{x}(t)$$
(2)

the full order Luenberger state observer is described by the following state equation:

$$\frac{d}{dt}\underline{\hat{x}}(t) = \underline{A}\underline{\hat{x}}(t) + \underline{B}\underline{u}(t) + \underline{K}\left[\underline{y}(t) - \underline{\hat{y}}(t)\right]$$

$$\underline{\hat{y}}(t) = \underline{C}\underline{\hat{x}}(t)$$
(3)

In the case of the drive system with elastic joint, the state vector of the drive system (1) was extended by the load torque value, to obtain the estimation of all mechanical state variables: system:

$$\underline{x} = \begin{bmatrix} \omega_1 & \omega_2 & m_s & m_o \end{bmatrix}^T \tag{4}$$

The motor electromagnetic torque and speed were used as input and output variables, respectively.

$$\underline{u} = m_e , \quad y = \omega_1 \tag{5}$$

Thus the state, control and output matrices of this extended Luenberger observer are following:

$$\underline{A} = \begin{bmatrix} -\frac{d}{T_{m1}} & \frac{d}{T_{m1}} & -\frac{1}{T_{m1}} & 0\\ \frac{d}{T_{m2}} & -\frac{d}{T_{m2}} & \frac{1}{T_{m2}} & -\frac{1}{T_{m2}}\\ \frac{1}{T_c} & -\frac{1}{T_c} & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} \frac{1}{T_{m1}} \\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

The gain matrix  $\underline{K}$  was determined using geneticgradient algorithm (GGA) with the following cost function:

$$\begin{aligned} F_{1 for \ T_{m2}=T_{m2}} &= \left\{ \sum_{1}^{n} \left( \left| m_{s} - m_{se} \right| \right) * \sum_{1}^{n} \left( \left| \omega_{2} - \omega_{2e} \right| \right) * \sum_{1}^{n} \left( \left| m_{o} - m_{oe} \right| \right) \right\}, \\ F_{2 for \ T_{m2}=0.5T_{m2}} &= \left\{ \sum_{1}^{n} \left( \left| m_{s} - m_{se} \right| \right) * \sum_{1}^{n} \left( \left| \omega_{2} - \omega_{2e} \right| \right) * \sum_{1}^{n} \left( \left| m_{o} - m_{oe} \right| \right) \right\}, \\ F_{3 for \ T_{m2}=2T_{m2}} &= \left\{ \sum_{1}^{n} \left( \left| m_{s} - m_{se} \right| \right) * \sum_{1}^{n} \left( \left| \omega_{2} - \omega_{2e} \right| \right) * \sum_{1}^{n} \left( \left| m_{o} - m_{oe} \right| \right) \right\}, \\ F &= \min\left( F_{1} * F_{2} * F_{3} \right) \end{aligned}$$

so the main attention was focused in the best estimation of all variables for different values of the mechanical time constant of the load machine. After the GGA optimisation procedure the gain matrix for the extended state observer of the drive system considered in the paper was adjusted:  $\underline{K} = [12 \text{ e-7 e-9 } 182].$ 

## 3.2 Mathematical model of the extended Kalman filter for two-mass drive system

The same extended state equations were used for the Kalman filter design. According to the theory of Kalman filtering, it was assumed that a system is disturbed with Gaussian white noises, which represent process and measurement errors (w(t), v(t)). The system is described by Eq.8:

$$\frac{d}{dt}\underline{x}(t) = \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t) + \underline{w}(t)$$

$$\underline{y}(t) = \underline{C}\underline{x}(t) + \underline{v}(t)$$
(8)

where:  $\underline{A}, \underline{B}, \underline{C}, \underline{x}, \underline{y}, \underline{u} - as(4) - (6).$ 

After discretisation of Eq. 8 with  $T_s$  sampling step, the state estimation using Kalman Filter algorithm is calculated by Eq.9:

$$\frac{\hat{x}(k+1/k+1)}{\hat{x}(k+1/k) + \underline{K}(k+1) \left[\underline{y}(k+1) - \underline{C}(k+1)\hat{x}(k+1/k)\right]^{(9)}}$$

where the gain matrix is obtained by the following numerical procedure:

$$\underline{P}(k+1/k) = \underline{A}(k)\underline{P}(k/k)\underline{A}(k)^{T} + \underline{Q}$$

$$\underline{K}(k+1) =$$

$$\underline{P}(k+1/k)\underline{C}(k+1)^{T} [\underline{C}(k+1)\underline{P}(k+1/k)\underline{C}(k+1)^{T} + \underline{R}]^{-1}$$

$$\underline{P}(k+1/k+1) =$$

$$[\underline{P}(k+1/k) - \underline{K}(k+1)\underline{C}(k+1)]\underline{P}(k+1/k)$$
(10)

with state and measurement covariance matrices  $\underline{O}$  and  $\underline{R}$ .

The suitable choice of covariance matrices is rather difficult task, usually solved by trial and error. In this paper the same genetic-gradient algorithm was used with the cost function (7). After the optimisation procedure these matrices are set to:  $Q = diag[0.25 \ 35 \ 3e-12 \ 1e4], R = diag[75].$ 

## 4. SIMULATION AND EXPERIMENTAL RESULTS OF MECHANICAL VARIABLES ESTIMATION

### 4.1 Simulations results

In simulation and experimental tests the motor was fed by the AC/DC power converter with switching frequency 5 kHz. The sampling step used for the rotor speed measurement in the open-loop system was 0.2 ms. In Fig.1 simulated transients of real and estimated torsional torque  $m_{s_2}$  load speed

 $\omega_2$ , load torque  $m_o$  for both estimators are presented, in the case of nominal drive parameters. Then the estimation errors caused by parameters mismatch in estimators' models were calculated and presented in the following figures. The estimation errors were calculated according to the Eq.11:

$$\Delta_j = \frac{\sum\limits_{i=1}^{N} \left| x_j - \hat{x}_j \right|}{N}; \qquad (11)$$

where:  $x_j$ - actual motor variable,  $\hat{x}_j$ - estimated variable, N- number of a sample.

Motor inertia, stiffness coefficient and load inertia were changed to check the sensitivity of Luenberger observer and Kalman filter. In Fig.2 an example of errors calculated according to Eq.11 for both observers are presented in the case of load inertia changes. It is seen that for the same range of parameter changes, the Kalman filter presents much lower estimation errors and better dynamics in the case of fast step changes of motor speed or load torque.



Fig. 1 Transients of the real and estimated torsional torque (a, d), load speed (b, e), load torque (c, f) for nominal drive parameters, obtained using Luenberger observer (a, b, c) and Kalman filter (d, e, f)



Fig. 2 Estimation errors of torsional torque (a), load speed (b) and load torque (c) for changed load inertia: from left: 50%, 75%, 100%, 150% and 200% of nominal value, obtained using Luenberger observer (a - c) and Kalman filter (d - f)



**Fig. 3** Transients of the real and estimated torsional torque (a, b), load speed (c, d), load torque (e, f) for changed drive parameter  $T_{m2}$ , obtained using Luenberger observer (a, c, e) and Kalman filter (b, d, f)



**Fig. 4** Estimation errors of torsional torque (a), load speed (b) and load torque (c) for changed stiffness coefficient: from left: 50%, 75%, 100%, 150% and 200% of nominal value, obtained using Luenberger observer (a - c) and Kalman filter (d - f)

In Fig.3 simulated transients of real and estimated torsional torque  $m_s$ , load speed  $\omega_2$ , load torque  $m_o$  for both estimators are presented, in the case of changing drive parameters ( $T_{m2} = 2T_{m2N} = 460ms$ ).

Next, other estimation errors are presented. In Fig.4 and Fig.5 the sensitivity to changes of stiffness coefficient and motor inertia are presented respectively.



**Fig. 5** Estimation errors of torsional torque (a), load speed (b) and load torque (c) for changed motor inertia: from left: 50%, 75%, 100%, 150% and 200% of nominal value, obtained using Luenberger observer (a - c) and Kalman filter (d - f)

#### 4.2 Experimental results

In Fig.6 the structure of the experimental set-up is presented. It is composed of a DC motor driven by a four-quadrant chopper. The motor is coupled to a load machine by an elastic shaft (a steel shaft of 5mm diameter and 600mm length). The moment of inertia can be varied by a flywheel, where the inertia ratio of motor to load varies from 0.125 to 8. The load machine is also a DC motor. The two motors have the nominal rating of 500W each. Speed and position of the motors are measured by incremental encoders (5000 pulses per rotation). The mechanical system has a natural frequency approximately 9.5 Hz. The control and estimation algorithms are implemented in digital signal processor using dSPACE software.



Fig. 6 The schematic diagram of experimental set-up

Both estimators of mechanical state variables were tested in the open-loop system, and the measured motor variables were compared to estimated ones. First, the system with the Luenberger observer was tested. A lot of experiments were carried out to check the system's performance. According to the defined cost function (7), the most interesting are transients obtained with changed inertia of the load side. They are presented in Fig.7, for low speed range.

In Fig.7 a-c, the real and the estimated motor speed are presented. For smaller value of load side inertia the results are poor. The estimated speed oscillates especially for change of the speed reference value. For nominal parameters of the system (Fig.7b) the Luenberger observer works very well, only small errors occur while changing disturbance torque. In Fig.7c the estimation results in the case of two times bigger load side inertia are presented. The responses are slower than in the case of decreasing of  $T_{m2}$  and they have no oscillations. In Fig.7 d-f, estimation errors between the real and the estimated speeds are presented. In Fig.7 g-i, the real electromagnetic and estimated torsional and disturbances torques are shown. Unfortunately, in the experimental set-up there is no measurement of these last two variables, so there is no possibility to check the accuracy of the estimation.

In Fig. 8 experimental results obtaining for Kalman filter are presented. The real and estimated load speed transients are presented in Fig.8 a-c. Despite of changing parameters of the system, results of the estimation are good. In Fig.8 g-i, the real electromagnetic and estimated torsional and disturbances torques are presented. In the case of Kalman Filter much smaller estimation errors are obtained in comparison with results for the Luenberger observer.



**Fig. 7** Transients of real and estimated load speed (a-c), its error (d-f), real motor and estimated torsional and load torques (g-i), for changed load inertia, obtained using Luenberger observer; from left: 50% (a, d, g), 100% (b, e, h) and 200% (c, f, i) of nominal value





**Fig. 8** Transients of real and estimated load speed (a - c), its error (d - f), real motor and estimated torsional and load torques (g - i), for changed load inertia, obtained using Kalman Filter; from left: 50% (a, d, g), 100% (b, e, h) and 200% (c, f, i) of nominal value

## 5. CONCLUSION

In this work the sensitivity analysis of mechanical variable estimation of the two-mass drive system has been done. Two different methods have been used, Luenberger Observer and Kalman Filter. The estimation errors for nominal and changed system parameter have been calculated. It was shown that results obtained for Kalman filter are much better than for Luenberger observer. For nominal parameters of the system the performances of the Luenberger Observer are quite good and this method is very often used in real system. But in the case of changing system parameters the estimated system's transients in dynamic state have relatively big errors, unacceptable in practical systems; the steady stay errors are equal to zero. The Kalman Filter provides much better results as Luenberger Observer for nominal as well as for changed system parameters. The estimation time of all variables is much faster and the filter is more robust to drive parameters changes.

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### BIOGRAPHIES

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