# FAULT STATE SELECTION IN THE POWER SYSTEM

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#### SUMMARY

This paper deals with the selection of fault states that are further assessed in order to work out operability of a power system. The selection of fault states is necessary for large power systems, where assessment of all possible fault states in terms of system operability would be extremely time-consuming because the number of all fault states can be in the case of complex networks very high. Therefore it is needed to reasonably reduce the number of fault states that are to be assessed. Three different approaches are described in this article. Their goal is to find a set of those faults states that are probable enough in real operation conditions. Only such fault states are then assessed in terms of power reliability and power system operability.

**Keywords:** analysis, reliability, power system, elements, probability, failure, fault state, limit state, node, reliability diagram, reliability parameters

### 1. INTRODUCTION

In order to decide on the operability of a power system we can use some of the criteria described in [2]. For complex power systems, it is not advisable to assess every single possible fault state of the system because number of all fault states can be extremely high so the evaluation of all these states would consume too much time. Hence it is a common practice to select only some of these states for further evaluation. Some of the methods used for the selection are described in this paper.

#### 2. FAULT STATE SELECTION IN THE POWER SYSTEM

I. Commonly used is e.g. a "(n-1) method" or "(n-2) method" which assume as really probable only those states that result from a failure of one and two elements, respectively. A concurrent failure of more than two elements is considered really improbable and thus power system states are not assessed for this case. For instance, using the "(n-2) method", it is necessary to evaluate  $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} = \frac{n^2 + n + 2}{2}$  different states. In a power system containing a large number of

elements, even this number of states needing evaluation is too high so it is necessary to limit this number by a certain value of probability of occurrence for such states. For example, all states with probability of occurrence lesser than  $10^{-3}$  will not be considered.

**II.** Other method for fault state selection is the following one:

Assume a system containing n elements in total. Each of these elements i = 1, 2, ..., n can find itself either in operation state with probability  $p_i$  or in fault state with probability  $q_i$ . It stands:

$$p_i + q_i = 1 \tag{2.1}$$

A system of n elements can find itself in one of  $N = 2^n$  different states. Each of these states can be generally described by a vector  $\mathbf{x}_k$ , k = 1, 2, ..., N, which contains n elements:

$$\mathbf{x}_{k} = (r_{1}, r_{2}, \dots, r_{n}) \tag{2.2}$$

Whereas  $r_i$  has either value 1 if the corresponding element i is operating, or value 0 if the element is in failure. Let us denote  $\mathbf{x}^{M}$  as the highest and  $\mathbf{x}^{m}$  as the lowest operating state:

$$\mathbf{x}^{M} = (1, 1, \dots, 1) \tag{2.3}$$

$$\mathbf{x}^{m} = (0, 0, \dots, 0) \tag{2.4}$$

The probability that the system finds itself in a state  $\mathbf{x}_k$  is given by:

$$h(\mathbf{x}_k) = \prod_{i=1}^n h(r_i)$$
(2.5)

where  $h(r_i)$  is the probability that an element i finds itself in state  $r_i$ . For  $r_i = 1$  will be  $h(r_i) = p_i$ , for  $r_i = 0$ will be  $h(r_i) = q_i$ . From it results:

$$f(\mathbf{x}^M) = \prod_{i=1}^n p_i \tag{2.6}$$

$$f(\mathbf{x}^m) = \prod_{i=1}^n q_i \tag{2.7}$$

A set of all states  $\mathbf{x}_k$  must be then divided into a subset of unacceptable states Q, i.e. states in which the power demand is not met in sufficient quality and quantity, and a subset of acceptable states P, i.e. states in which the power demand is met in sufficient quality and quantity. This division depends on the chosen operability criteria. The probability of insufficient power supply in the power system is given by

$$Q_s = \sum_k h(\mathbf{x}_k), \quad \mathbf{x}_k \in Q$$
(2.8)

and the probability of sufficient power supply by

$$P_s = 1 - Q_s = \sum_k h(\mathbf{x}_k), \quad \mathbf{x}_k \in P$$
(2.9)

But the outlined process of power system reliability assessment, resulting from relationships (2.5), (2.8), (2.9), necessitates the evaluation of  $N = 2^n$  different states, which for instance in a system comprising 100 elements means  $2^{100} = 1,267 \cdot 10^{30}$  states. Therefore we carry on this way:

At first, we divide the original set of N states  $\mathbf{x}^m \leq \mathbf{x}_k \leq \mathbf{x}^M$  into three subsets:

- 1. subset P acceptable states (the system is operable)
- 2. subset Q unacceptable states (the system is inoperable)
- 3. subset R states so far unclassified

In order to do this, it is necessary to find so-called upper and lower limit state  $\mathbf{x}^{R}$  and  $\mathbf{x}^{r}$  so that stands:

 $\begin{array}{l} \text{if } x_k \geq x^R \text{ then } x_k \in P \\ \text{if } x_k < x^r \text{ then } x_k \in Q \\ \text{if } x^r \leq x_k < x^R \text{ then } x_k \in R \end{array}$ 

After that the actual value of  $Q_s$  according to (2.8) and actual value of  $P_s$  according to (2.9) can be calculated. It can be written in short as:

$$Q_s = h(Q), \quad P_s = h(P) \tag{2.10}$$

Furthermore stands:

$$h(R) = 1 - h(P) - h(Q) \tag{2.11}$$

If  $h(R) \leq \varepsilon$ , where  $\varepsilon$  is an arbitrarily chosen small positive number, it means there is only small probability that the system shifts to some of states  $x_k \in R$  and thus the calculation can be considered finished.

If  $h(R) > \varepsilon$  it is necessary to repeat the whole calculation with the set R as original.

For finding the limit states  $\mathbf{x}^{R}$  and  $\mathbf{x}^{r}$ , e.g. method of maximum flow in the transfer network can be used. If the maximum flow through graph  $S_{k}$  under certain state of network  $\mathbf{x}_{k}$  is lower than the total power load of network S (as a result of either insufficient transfer capacity or insufficient generating capacity), then this state belongs to the subset of states Q, and, on the contrary, if  $S_{k} > S$ , then the state  $\mathbf{x}_{k}$  belongs to the subset P.

When evaluating the limit state  $\mathbf{x}^{R}$ , we start from the state  $\mathbf{x}^{M}$  according to (2.3) for which we work out the maximum flow through graph S<sup>M</sup>. Power flows in individual elements are labeled  $f_i$ ,  $i = 1 \div n$ . The state  $\mathbf{x}^R$  is then given by:

$$\mathbf{x}^{R} = (r_{1}^{R}, r_{2}^{R}, \dots, r_{n}^{R})$$
(2.12)

where 
$$r_i^R = 0$$
 if  $f_i = 0$   
 $r_i^R = 1$  if  $0 < f_i \le c_i$ 

When evaluating the limit state  $\mathbf{x}^r$  we start from the state  $\mathbf{x}^M$  according to (2.3) in which we eliminate  $i^{th}$  element (we put  $c_i = 0$ ) and work out the maximum flow  $S_i$ . The  $i^{th}$  element  $r_i^r$  of the state vector  $\mathbf{x}^r$  is then either  $r_i^r = 1$  if  $S_i < S$ , or  $r_i^r = 0$ , if  $S_i$  $\geq S$ . We carry out the same procedure for all  $i = 1 \div n$ , hence we obtain the state vector  $\mathbf{x}^r$  in this form:

$$\mathbf{x}^{r} = (r_{1}^{r}, r_{2}^{r}, \dots, r_{n}^{r})$$
(2.13)

If we know the vectors  $\mathbf{x}^r$ ,  $\mathbf{x}^R$ , then according to (2.11) the probability h(R) can be calculated, i.e. the probability that the system finds itself in some of so far unclassified states  $\mathbf{x}_k$  of the set R. If  $h(R) \leq \varepsilon$ , then the iteration process stops and reliability parameters can be worked out. If  $h(R) > \varepsilon$  or if the set R is desired empty ( $\mathbf{x}^r = \mathbf{x}^R$ ), then the iteration process cannot be stopped and a new division of the state  $\mathbf{x}_k \in R$  into acceptable ( $\mathbf{x}_k \in P$ ) and unacceptable ( $\mathbf{x}_k \in Q$ ) states has to be performed. This can be done by the following procedure:

At first, the set R of the states  $\mathbf{x}_k$  is divided into subsets  $R_j$  in such a way that states  $\mathbf{x}_k \in R_j$  meet the condition that time, if

$$\mathbf{x}_{k} = (r_{1}, r_{2}, \dots, r_{n}) \in R_{j}$$

$$r_{j}^{r} \leq r_{j} < r_{j}^{R}$$

$$r_{i} \geq r_{i}^{R} \quad \text{for } i < j$$

$$r_{i} \geq r_{i}^{r} \quad \text{for } i > j$$

$$(2.14)$$

This way we obtain several disjunctive sets  $R_j$ . In each of them the values of limit states  $\mathbf{x}^{R}(R_j)$ ,  $\mathbf{x}^{r}(R_j)$  can be found using the procedure described above. Then for every j for which the set is non-empty stands:

 $\begin{array}{l} \text{if } \mathbf{x}_k \in R_j, \, \mathbf{x}_k \geq \mathbf{x}^R(R_j), \, \text{then } \mathbf{x}_k \in P_j \in P, \\ \text{if } \mathbf{x}_k \in R_j, \, \mathbf{x}_k \leq \mathbf{x}^r(R_j), \, \text{then } \mathbf{x}_k \in Q_j \in Q, \\ \text{if } \mathbf{x}_k \in R_j, \, \mathbf{x}^r(R_j) \leq \mathbf{x}_k \leq \mathbf{x}^R(R_j), \, \text{then } \mathbf{x}_k \in R. \end{array}$ 

If the states of set Q are known (either all, if the set R was exhausted, or just their part, if the condition  $h(R) \le \varepsilon$  was met), then it is possible to evaluate the probability of insufficient power supply in the system according to formula (2.8). A number of states  $\mathbf{x}_k$  in set Q can be very high, so the calculation according to (2.8) would be very demanding. Furthermore, the calculation must be carried out in each iteration. Thus, for easier evaluation of  $Q_s$ , we carry out certain restructuring

of states  $\mathbf{x}_k$  within set Q by dividing the set Q into disjunctive sets  $Q_m$ . Then stands:

$$Q_s = h(Q) = \sum_m h(Q_m) \tag{2.16}$$

The sets  $Q_m$  must be disjunctive, i.e. each value of the state vector  $\mathbf{x}_k \in Q$  complies with conditions of only one of the sets  $Q_m$ .

Division of the set Q into sets  $Q_m$  is given by the following condition:

$$\mathbf{x}_{k} = (r_{1}, r_{2}, \dots, r_{m}) \in Q_{m}$$

$$r_{m} < r_{m}^{r} \qquad (2.17)$$

$$r_{i} \ge r_{i}^{r} \qquad \text{for } i < m$$

Each set of states  $\mathbf{x}_k \in Q_m$  is unacceptable because of the fact that its  $m^{th}$  element is in failure. The remaining elements can find themselves in any state under the condition that elements i < m with  $r_i^r$ = 1 must not be in failure. If we divide the set Q into sets  $Q_m$  using the described method, the probability  $h(Q_m)$  can be calculated easily.

$$h(Q_m) = q_m \prod_{i < m} h(r_i \ge r_i^r)$$
(2.18)

where  $h(r_i \ge r_i^r) = 1$  if  $r_i^r = 0$  $h(r_i \ge r_i^r) = p_i$  if  $r_i^r = 1$ 

In formula (2.11), it is necessary to work out also the value of h(P) in each iteration. It can be done using this relationship:

$$h(P) = \prod_{i} h(r_i \ge r_i^R)$$
(2.19)

where  $h(r_i \ge r_i^R) = 1$  if  $r_i^R = 0$  $h(r_i \ge r_i^R) = p_i$  if  $r_i^R = 1$ 

Relationship (2.19) may be used also for calculation of probabilities  $h(P_j)$ , where  $P_j$  are partial sets of states  $\mathbf{x}_k \ge \mathbf{x}^R(R_j)$  according to (2.15). Then

$$h(P) = \sum_{j} h(P_j) \tag{2.20}$$

**III.** A third method for selection of probable fault states is the Sinchugov's method. It is suitable especially for reliability assessment of those power grids containing substations of relatively complex configuration. The method takes into account mutual dependence of failures of individual elements in the power system. Based on this method in connection with the Ford-Fulkerson algorithm for maximum flow through a graph, which is used as a power system operability criterion, a computer program has been built, debugged and put into use. The program solves the problems of fault-free power supply in load nodes and in the power system as a whole. We

will describe here only the basic principles of the whole procedure. A mathematical model for a method of fault state selection results from so-called reliability diagram of network that is achieved by transformation of the original network configuration. The reliability diagram is composed of nodes interconnected by branches. Under the term "node" we understand a set of mutually connected equipment (e.g. power lines, cables, bus-bars, power transformers, sources, disconnectors, power instrument transformers, discharge arresters, appliances etc.) as far as to circuit breaker inputs. Thus, circuit breakers are not comprised in nodes. During a failure of any piece of equipment the node comprises, the whole node must be disconnected from the power system by all circuit breakers that border this node, i.e. the circuit breakers through which the node is connected to other nodes of the power system. Under the term "branch" of reliability diagram we understand a circuit breaker.

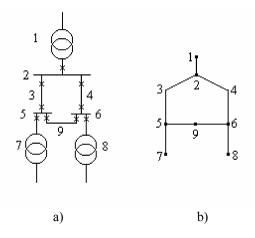


Fig. 1 Diagram of a power system

In Fig.1a) is a diagram of a power system's part comprising 9 elements (3 transformers, 3 power lines and 3 buses) and 9 circuit breakers.

Each circuit breaker interconnects 2 elements. In Fig.1b) is a reliability diagram of the system, where nodes represent the system's elements and branches represent the circuit breakers. The reliability diagram is essential for fault state selection. At first, it is necessary to establish reliability characteristics for nodes and branches of reliability diagram. For instance, intrinsic failure rate  $\lambda$  for a certain node equals sum of failure rates  $\Sigma \lambda_i$  for all components forming the node (as if all components were arranged in series). Similarly, failure rate for circuit breaker can be evaluated; it is furthermore divided into intrinsic (or short-circuit) failure rate of a circuit breaker and breaker malfunction rate during switching-off the faults in nodes connected to the circuit breaker.

Based on reliability parameters established in such a way (i.e. intrinsic failure rate, mean time of failure repair, revision rate and planned repair rate, mean time of planned shut-downs, operating mean time, etc.), generalized reliability parameters can be worked out. They are overall parameters of the circuit breaker and node reliability and take into account mutual coupling between nodes and circuit breakers, and also respect the possibility of failure propagation from one node to another.

For instance, according to Fig.1b), an outage of node 9 can occur in these cases:

- 1. failure in node 9
- 2. intrinsic (short-circuit) failure of circuit breaker 5-9 or 6-9
- 3. failure in node 5 or 6 and concurrent malfunction of circuit breaker 5-9 and 6-9 respectively
- 4. intrinsic failure of circuit breaker 3-5 or 5-7 and/or 4-6 or 6-8 and concurrent malfunction of circuit breaker 5-9 and/or 6-9
- failure in node 3 and concurrent malfunction of circuit breakers 3-5 and 5-9, or failure in node 7 and concurrent malfunction of circuit breakers 5-7 and 5-9, or failure in node 4 and concurrent malfunction of circuit breakers 4-6 and 6-9, or failure in node 8 and concurrent malfunction of circuit breakers 6-8 and 6-9

Other possibilities, i.e. intrinsic failure or malfunction of more than two circuit breakers are regarded as improbable. General reliability parameters for circuit breakers are worked out similarly.

If we have worked out generalized reliability parameters, we define so-called calculation sections of 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> degree. Calculation sections have to be defined for each load node, in which we calculate reliability. For instance, a calculation section of 1<sup>st</sup> degree (related to a specified load node) is a part of reliability diagram, comprising one or more nodes interconnected by branches, where outage of any single node or branch leads to the case when power demand of one load node, or power demand of more load nodes concurrently, is not met. A calculation section of 2<sup>nd</sup> degree (related to a specified load node) is a part of reliability diagram, comprising two nodes whose concurrent outages lead to the case when power demand of the specified load node, or power demand of more load nodes concurrently, is not met. Similarly defined is a calculation section of  $3^{rd}$  degree. It is obvious that nodes and branches of calculation sections of  $1^{st}$ degree do not group the nodes and branches of calculation sections of  $2^{nd}$  degree, and nodes and branches of calculation sections of 3rd degree do not group the nodes and branches of calculation sections of 1<sup>st</sup> and 2<sup>nd</sup> degree. We add to the group of calculation sections of 2<sup>nd</sup> degree also those node whose mutual failure dependence is pairs documented statistically (e.g. double circuit line on the same transmission towers).

Calculation sections must be searched for systematically, using repeatedly some of the power system operability criteria.

Reliability parameters of individual calculation sections can be then calculated (failure rate, mean time of outage, failure probability, etc.). Reliability parameters of load node, i.e. outage rate, insufficient power supply probability, mean (expected) value of unsupplied load, etc. are then worked out from reliability parameters of all calculation sections corresponding to the specified load node.

### 3. CONCLUSION

# 1<sup>st</sup> method

When the number of elements in the power system is large, then the number of states needing evaluation is too high, so it is necessary to further limit this number by a certain value of probability of occurrence for such states. In other words, calculation of the network with a large number of elements using this method would be very complicated.

# 2<sup>nd</sup> method

The described method of power system reliability assessment demands evaluation of  $2^n$  different states, which means a large number of states. That's why we further simplify this criterion. After simplification, the states of set Q are known (either all or only part of them). Then the insufficient power supply probability can be worked out. But there can be a great deal of states  $x_k$  in the set Q, so the calculation then can be also very demanding.

## 3<sup>rd</sup> method

This method is suitable especially for reliability assessment of those power grids containing substations of relatively complex configuration. The method takes into account mutual dependence of failures of individual elements in the power system. A computer program based on this method has been built, which solves the problems of fault-free power supply in load nodes and in the power system as a whole. It significantly speeds up and makes easier the calculation that would otherwise have to be carried out manually.

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### BIOGRAPHY

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