SIGNAL FLOW GRAPH THEORY APPLICATION FOR OBTAINING THE STATE SPACE MODEL FROM BOND GRAPH MODEL WITH DERIVATIVE CAUSALITY

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SUMMARY

A new approach for deriving the state space equations from the causal bond graph with derivative causality, on the basis of equivalent signal flow graph, is given. By following "the paths of traveling" of two general bond graph variables (effort e and flow f) through the causal bond graph, the equivalent signal flow graph is obtained. On the basis of that signal flow graph, the system of equations with the first derivatives of state variables which are implicitly expressed, are formed. After their minor arranging the state space model is obtained.

Keywords: *bond graph, signal graph, state space model*

1. INTRODUCTION

The state space model deriving from the bond graph model is very important in the practical engineering. Thus, the strong connection between bond graph models of systems and all the analysis and design methods based on the state space model is realized. That is why it is easy to understand the importance of systematic procedures for obtaining the state space model from the bond graph.

In bond graphs theory, a significant attention is paid to causality. Causality was introduced to find out whether it is possible to calculate all model variables after presenting a system model as a bond graph. By an undivided definition of causes and consequences one can clearly see how to calculate any model variable. It is obvious that a consequence cannot be determined, unless all its causes including the main sources of model energy causes are known. It is also necessary to be aware of their mutual influence. So causality clearly shows the directions of the source energy propagation to the latest in bond graph. Once causality is determined (procedure described in [1-5]), the equations for determination of effort and flow of each bond in a bond graph can be derived from a bond graph. It is of interest to derive equations which describe system state space model. In 1988, *Breedveld* proposed the procedure for obtaining the relations between effort and flow for every bond graph element considering causality stroke position (The elements are: Resistive element (R element), Flow storage element (C element), Effort storage element (I element), Transformer (TF element), Gyrator (GY element), Effort source (S_e element), Flow source (S_f element), Series junction (1 junction) and Parallel junction (0 junction)). A desired equation system can be determined by ordering initial equation system [1]. In 1989, *Cornet* and *Lorenz* proposed the procedure based on two additional equation systems [6]. The first one, the "head queue", is organized as FIFO (first input first output), and the second one, the "tail stack", is organized as LIFO (last input first output). The desired (ordered) equation system is determined by calculating "forward" variables of the first list and "reverse" of the second one, while following causality propagation in bond graph [6]. However, a special approach to this problem, based on using signal graph theory as an indirect tool, is put forward in [7, 8]. The systematic procedure for obtaining the state space model based on the bond graph model and the corresponding signal graph, presented in [7] is limited to the specific case where the bond graph contains only integral causality. The state space model may be obtained directly from the signal graph without any transformation of the equations. However, the presence of derivative causality on any C or I element points out the existence of dynamical dependence among the state space variables so that it requires the additional transformations of the equations obtained from the signal graph which is equivalent to initial bond graph [8]. The other methods for obtaining the state space model from the bond graph model are reduced to the writing of the equations for each element and connection in the bond graph, and their additional arranging [1, 6]. The number of necessary transformations of initial equations is increased with the bond graph complexity increasing. Then, the application of these methods become rather complicated.

In this paper, the procedure from [7, 8] is generalized by enhancing its application on, as well, the causal bond graph models in which there is derivative causality beside the integral one. First, an actual example will be elaborated, and then, on its basis, the generalization in the form of systematic procedure of obtaining the state space model based on the bond graph model and the equivalent signal flow graph will be presented. The described procedure is then applied to the bond graph models containing both integral and derivative causality.

2. SYSTEMATIC PROCEDURE FOR OBTAINING EQUIVALENT SIGNAL FLOW GRAPH BASED ON BOND GRAPH

Let us consider a bond with causality as shown in Figure 1, where energy flow direction, that is half arrow, is omitted. Considering well known convention, causality stroke is oriented in the direction of the effort propagation, while flow variable travels in the opposite direction [6,9]. Effort propagation is denoted by a dashed line (-----), and flow propagation is denoted by a dotted line (.....), as shown in Figure 1. A bond graph clearly indicates that their paths are mutually dependent and make intersections in specific places (GY elements). Effort path begins where the flow path ends, and vice versa, in ending R, C and I elements, which can be described by appropriate mathematical equations of junction.

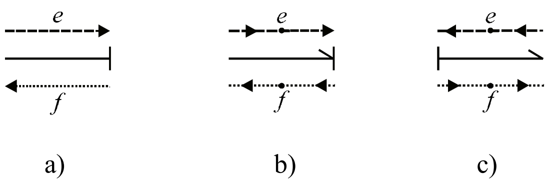

Figure 1. Signal flow through bond element

A following transformation is introduced in order to define a desired connection between the bond graph and the corresponding signal, graph which represents the way effort, and flow travel. Every bond can be described as two nodes (in corresponding signal graph), the first one describes the effort on the bond (e - node) and the second one describes the flow on the bond (f - node), as shown in Figure 1b and 1c. Two signal graph branches (one for the flow and the other one for the effort), which connect corresponding nodes, are parallel to the bond but arrow directions are opposite and are always such that the effort branch is directed towards the causality stroke, and the flow branch is directed backwards.

Using the same principle, we can define transformations for each element, source and junction of bond graph. For instance, R element has its equivalent element in the corresponding signal graph, as shown in Figure 2. Obviously, when R element causality is assigned in such a manner, the rest of the bond graph will act as an effort source, and flow can be determined using the element itself as $f = \frac{1}{R}e$. Equivalent R element in the corresponding signal graph (Figure 2) describes the connection between effort and flow as a branch with a gain $\frac{1}{R}$

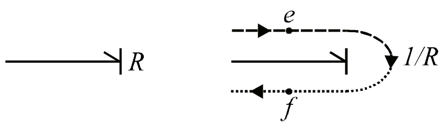

Figure 2. R bond graph element and its equivalent signal graph

If R element causality is represented as shown in Figure 3, then equivalent R element of a corresponding signal graph can be represented as shown in the same figure. For such R element causality, the rest of the bond graph acts as a flow source, and effort can be determined using the element itself as $e = Rf$.

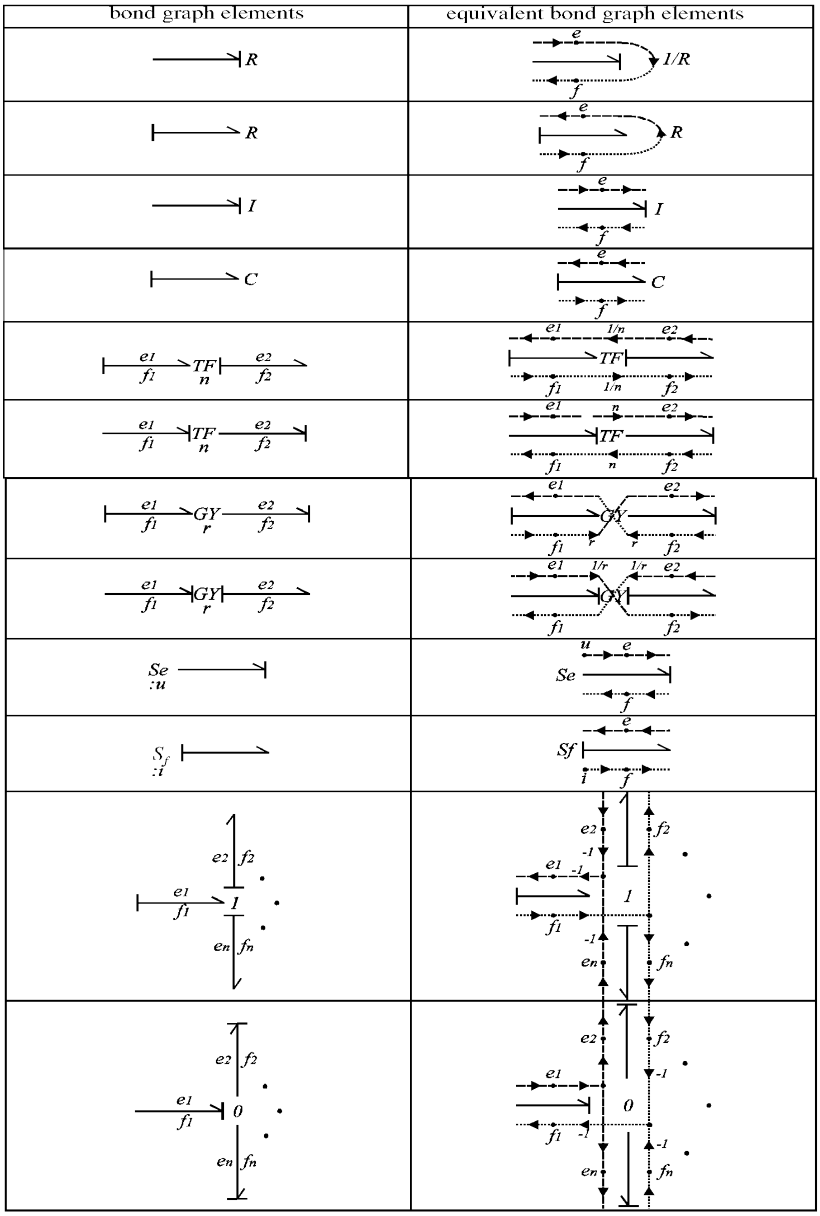

Table 1. Bond graphs and their corresponding signal graphs

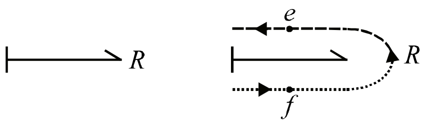

Figure 3. R bond graph element and its equivalent signal graph

Basic elements, sources and junctions of a bond graph and their equivalent elements, sources and junctions of a corresponding signal graph, which enable direct signal graph obtaining based on bond graph are shown in Table 1.

Finally, if direction of effort ("1" junction) or flow ("0" junction) is equivalent to a half-arrow direction on the bond then the branch gain is +1, and if not then the branch gain is -1. For branches without gain associated to it, it is considered to be +1.

For a specific bond graph, variables of interest are: C elements effort and I elements flow. These are at the same time state variables of a corresponding state space model. The nodes of equivalent signal graph variables are of special interest for the proposed procedure.

Taking the bond graph model as a starting point, the corresponding signal flow graph can be obtained by applying the following systematic procedure [7]:

1. Marking the causal orientation of bonds in bond graph model (adding causality strokes to bond graph model), as described in [1], [6] and [10].
2. The equivalent signal flow graph obtaining,
   - each *R* bond graph element is replaced by the equivalent *R* element from Table 1,
   - each *C* bond graph element is replaced by the equivalent *C* element from Table 1,
   - each *I* bond graph *I* element is replaced by the equivalent *I* element from Table 1,
   - each TF and GY are replaced by the equivalent TF and GY from Table 1,
   - every *"1"* and *"0"* junctions of the bond graph model are replaced by the equivalent *"1"* and *"0"* junctions from Table 1,
   - S_e and S_f sources are replaced by the equivalent elements (*u* and *i* - nodes) from Table 1,
   - The signal flow graph is made where nodes are sources, effort and flow of a bond graph.

All this should be linked to produce wholeness, i. e. the signal flow graph whose nodes, sources, efforts and flows are in the bond graph.

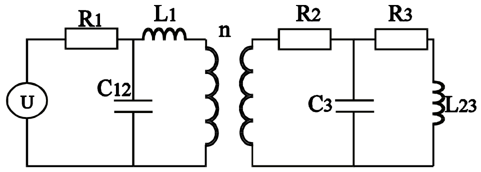

Figure 4. Electrical system

This procedure is illustrated by the example of the electric system in Figure 4, whose bond graph is shown in Figure 5, and the equivalent signal flow graph in Figure 6.

In this bond graph number 5 is intentionally omitted while numerating in order to obtain the same mark for the state variables as in the following example.

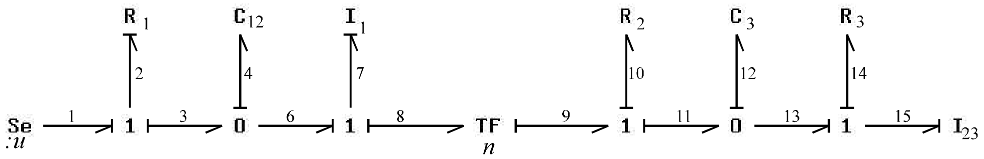

Figure 5. Bond graph model of electrical system from Fig. 4

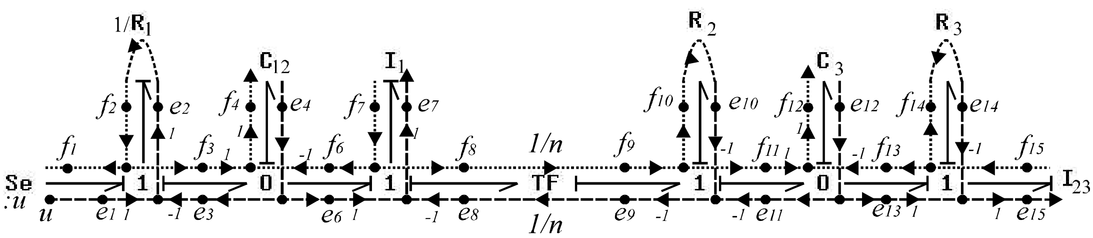

Figure 6. Signal flow graph representation of bond graph from Fig. 5

3. OBTAINING STATE SPACE MODEL FOR CONSIDERED ELECTRICAL SYSTEM

Consider signal flow graph shown in Figure 6 obtained from the bond graph model of the electrical system in Figure 4. Obviously, there is only an integral causality. Let us choose significant effort variables on *C* elements and flow variables through *I* elements. These will also be the state variables in derived state space model. The nodes corresponding to these variables and sources in the bond graph are the source nodes ($e_4, f_7, e_{12}, f_{15}, u$) and the nodes corresponding to their first derivatives (multiplied by the corresponding factor, respectively), i. e. by the flow through C elements and the efforts on I elements are purely sink nodes (f_4, e_7, f_{12}, e_{15}). Due to this, the state space model of the considered system is obtained by finding all the paths and gains between source and sink nodes, directly from Figure 6.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

where:

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{R_1 C_{12}} & -\frac{1}{C_{12}} & 0 & 0 \\ \frac{1}{L_1} & -\frac{R_2}{L_1 n^2} & -\frac{1}{L_1 n} & 0 \\ 0 & \frac{1}{nC_3} & 0 & -\frac{1}{C_3} \\ 0 & 0 & \frac{1}{L_{23}} & -\frac{R_3}{L_{23}} \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{R_1 C_{12}} & 0 & 0 & 0 \end{bmatrix}; \mathbf{x} = [e_4 f_7 e_{12} f_{15}]; \mathbf{u} = [u]$$

This state space model can be determined because there is no derivative causality in the bond graph model.

Now the electrical circuit in Figure 4 is modified by establishing the parallel junction of two capacitors C_1 and C_2 instead of capacitors C_{12} (so, $C_{12} = C_1 + C_2$) and the serial junction of two inductors L_2 and L_3 instead of inductor L_{23} (so, $L_{23} = L_2 + L_3$) as shown in Figure 7.

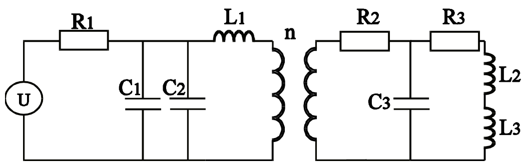

Figure 7. Modified electrical system in Fig. 4

Then the following is to be done,

1) Drawing the bond graph model of this system and determining the position of the causal strokes (Figure 8).

2) Drawing its equivalent signal flow graph on the basis of the procedure presented in Part 2 (Figure 9). It may be seen that in elements C_2 and I_3 there is derivative causality, which makes nodes e_{16} and f_5 be the source ones, and f_{16} and e_5 be sink ones (contrary to the case when in these elements there is integral causality). Now (as it can be seen in Figure 8) nodes $u, e_4, f_7, e_{12}, f_{15}, f_5$ and e_{16} are purely source ones, and nodes f_4, e_7, f_{12}, e_{15}, e_5 and f_{16} are purely sink.

3) From the signal flow graph in Figure 9, choose "significant" nodes (e_4, f_7, e_{12}, f_{15}) regardless the source nodes corresponding C_2 and I_3 elements (f_5 and e_{16}). Therefore, the state vector is $\mathbf{x} = [e_4 f_7 e_{12} f_{15}]$, the control vector is $\mathbf{u} = [u]$, and variables f_5 and e_{16} will not be state variables but dynamically dependent variables with state variables e_4 and f_{15} respectively. The state variables derivatives and variables e_5 and f_{16} derivatives are expressed in the function of the variables corresponding to the source nodes.

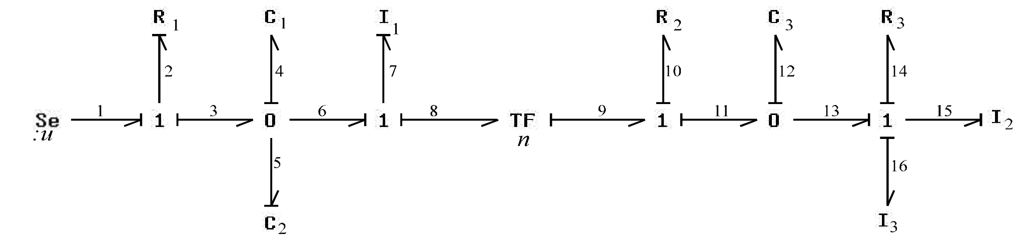

Figure 8. Bond graph model of system in Fig. 7

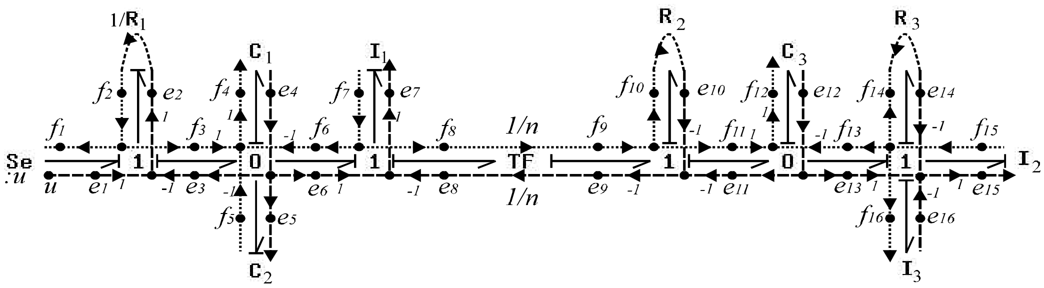

Figure 9. Equivalent signal flow graph obtained on the basis of bond graph in Fig. 8

$$\begin{aligned}
\frac{de_4}{dt} &= \frac{1}{C_1} f_4 = \frac{1}{C_1}\left(-\frac{1}{R_1} e_4 - f_7 - f_5 + \frac{1}{R_1} u\right) \\
\frac{df_7}{dt} &= \frac{1}{L_1} e_7 = \frac{1}{L_1}\left(e_4 - \frac{R_2}{n^2} f_7 - \frac{1}{n} e_{12}\right) \\
\frac{de_{12}}{dt} &= \frac{1}{C_3} f_{12} = \frac{1}{C_3}\left(\frac{1}{n} f_7 - f_{15}\right) \\
\frac{df_{15}}{dt} &= \frac{1}{L_2} e_{15} = \frac{1}{L_2}\left(e_{12} - R_3 f_{15} - e_{16}\right)
\end{aligned} \tag{2}$$

$$\begin{aligned}
e_5 &= e_4 \\
f_{16} &= f_{15}
\end{aligned} \tag{3}$$

4) By applying (3), f_5 and e_{16} are expressed as the following,

$$\begin{aligned}
f_5 &= C_2 \frac{de_5}{dt} = C_2 \frac{de_4}{dt} \\
e_{16} &= L_3 \frac{df_{16}}{dt} = L_3 \frac{df_{15}}{dt}
\end{aligned} \tag{3a}$$

5) By replacing f_5 and e_{16} from (3a) to equation system (2) and by explicitly expressing the derivative of variables e_4, f_7, e_{12} and f_{15} as a function of these variables and input variable u, the state space model is obtained in the form:

$$\begin{aligned}
\frac{de_4}{dt} &= \frac{1}{C_1 + C_2}\left(-\frac{1}{R_1} e_4 - f_7 + \frac{1}{R_1} u\right) \\
\frac{df_7}{dt} &= \frac{1}{L_1}\left(e_4 - \frac{R_2}{n^2} f_7 - \frac{1}{n} e_{12}\right) \\
\frac{de_{12}}{dt} &= \frac{1}{C_3}\left(\frac{1}{n} f_7 - f_{15}\right) \\
\frac{df_{15}}{dt} &= \frac{1}{L_2 + L_3}\left(e_{12} - R_3 f_{15}\right)
\end{aligned} \tag{4}$$

Obviously, the result is the same because it is the matter of the same electrical circuit with the same state variables. The only change is derivative causality in the bond graph model.

4. SYSTEMATIC PROCEDURE

The generalized systematic procedure for obtaining the state space model based on the bond graph model and the corresponding signal flow graph is easily determined by following procedure used for electrical circuit in Figure 7.

1. Drawing the bond graph model of the considered system and determining the positions of causal strokes
2. Drawing equivalent signal flow graph on the basis of the procedure obtained in part 2.
3. Choosing “significant” nodes, i. e. The state vector whose components are in fact the components of $\mathbf{e}_{\mathbf{C_i}}$ and $\mathbf{f}_{\mathbf{L_i}}$. Each sink node is expressed as a function of each source nodes ($\mathbf{u}, \mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}, \mathbf{f}_{\mathbf{C_d}}, \mathbf{e}_{\mathbf{L_d}}$), by using the determined graph as follows,

$$\begin{aligned}
\dot{\mathbf{e}}_{\mathbf{C_i}} &= \mathbf{C_i}^{-1}\mathbf{f}_{\mathbf{C_i}} = \mathbf{C_i}^{-1}\mathbf{f_1}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}, \mathbf{f}_{\mathbf{C_d}}, \mathbf{e}_{\mathbf{L_d}}, \mathbf{u}\right) \\
\dot{\mathbf{f}}_{\mathbf{L_i}} &= \mathbf{L_i}^{-1}\mathbf{e}_{\mathbf{L_i}} = \mathbf{L_i}^{-1}\mathbf{f_2}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}, \mathbf{f}_{\mathbf{C_d}}, \mathbf{e}_{\mathbf{L_d}}, \mathbf{u}\right)
\end{aligned} \tag{5a}$$

$$\begin{aligned}
\mathbf{f}_{\mathbf{C_d}} &= \mathbf{C_d}\frac{d\mathbf{e}_{\mathbf{C_d}}}{dt} = \mathbf{C_d}\frac{d}{dt}\left[\mathbf{f_3}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}\right)\right] \\
\mathbf{e}_{\mathbf{L_d}} &= \mathbf{L_d}\frac{d\mathbf{e}_{\mathbf{L_d}}}{dt} = \mathbf{L_d}\frac{d}{dt}\left[\mathbf{f_4}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}\right)\right]
\end{aligned} \tag{5b}$$

where:

$\mathbf{u}$ - the input vector
$\mathbf{e}_{\mathbf{C_i}}$ - the effort vector on *C* elements with integral causality
$\mathbf{f}_{\mathbf{L_i}}$ - the flow vector through *I* elements with integral causality
$\mathbf{f}_{\mathbf{C_d}}$ - the flow vector through *C* elements with derivative causality
$\mathbf{e}_{\mathbf{L_d}}$ - the effort vector on *I* elements with derivative causality
$\mathbf{C_i}$ -the diagonal matrix of *C* elements parameters with integral causality
$\mathbf{C_d}$ - the diagonal matrix of *C* elements parameters with derivative causality
$\mathbf{L_i}$ - the diagonal matrix of *I* elements parameters with integral causality
$\mathbf{L_d}$ - the diagonal matrix of *I* elements parameters with derivative causality

4. Replacing the expressions for $\mathbf{f}_{\mathbf{C_d}}$ and $\mathbf{e}_{\mathbf{L_d}}$ from (5a) in (5b).
5. From the considered system expressing explicitly,

$$\begin{aligned}
\dot{\mathbf{e}}_{\mathbf{C_i}} &= \mathbf{f}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}, \mathbf{u}\right) \\
\dot{\mathbf{f}}_{\mathbf{L_i}} &= \mathbf{g}\left(\mathbf{e}_{\mathbf{C_i}}, \mathbf{f}_{\mathbf{L_i}}, \mathbf{u}\right)
\end{aligned}$$

which features the state space model of the considered system.

It may be seen that this procedure, in the case of the bond graphs with derivative causality, still requires slight arranging of the obtained equations, while in the presence of only integral causality, it is not necessary.

5. CONCLUSION

In this paper, a new, systematic procedure for the state space system model obtaining based on the signal flow graph representation of the bond graph model is proposed. The signal flow graph clearly indicates the paths effort and flow variable travel through the bond graph, which allows to simply derive functional dependence between any two variables in the bond graph model. By selecting state variable as the efforts on *C* elements an the flows through *I* elements with integral causality and by finding the gains for paths between all "significant" nodes in the signal flow graph, the state space system may be obtained. The proposed procedure may be used for an arbitrary complex bond graph model by using, for instance, Mason rules for gains calculation. The simplicity of this procedure and the universality of application of the bond graphs on different physical systems, on which it is based, provide its usage in a wide area of application.

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BIOGRAPHIES

Dragan Antic was born in Vranje, Serbia, in 1963. He received the B.Sc., M.Sc., and Ph.D. degrees from the Faculty of Electronic Engineering, University of Nis, Serbia, in 1987, 1991, and 1994, respectively. He is now an Associate Professor at that faculty, where teaches courses in modelling and simulation and control system theory. He is the author and coauthor of a large number of papers. His research interests are modelling and simulation, variable structure systems, fuzzy sliding mode control, control of electric drives and industrial processes.

Lela Stankovic was born in Prokuplje, Serbia, in 1970. She received the B.Sc., and M.Sc. degrees from the Faculty of Electronic Engineering, University of Nis, Serbia, in 1994 and 1999, respectively. She is now a Research assistant at that faculty, where she is writing her Ph.D. thesis. Her research interests are bond graph modelling and simulation.