CONTRIBUTION TO THE COMPENSATION OF A NONLINEAR LOAD

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SUMMARY

In the article the method of optimised design of the passive LC type compensator for uniform line is described. The load at the end of the line can be linear, or non-linear, e.g. a semi-conductor converter. In the first case the currents and voltage in the line are harmonic, in the second case harmonic high orders are introduced into the line. These harmonics of higher orders increase power losses in the line. The article shows that a long line can be significant for simulating and designing a compensator.

Keywords: *compensation of line, non-linear power consumer, uniform line*

1. INTRODUCTION

Calculation of the capacity of power factor capacitor and inductance of balancing coils for a three-phase symmetrical load is known, if the line can be approximated as a circuit with lumped parameters. If we simulate line as a circuit with distributed parameters, the course of the current along the line is rather complicated and the calculation of the capacity of power factor capacitors is no more an elementary matter. The problem can be complicated by the fact that the load at the end of the line is non-linear (e.g. controlled thyristor) and drawn currents contain the harmonics of higher orders. Our paper is devoted to the optimal design of passive compensation.

2. ALGORITHM OF THE CALCULATION

2.1 Linear load

Let us consider a symmetrical three–phase linear load connected by a three-phase uniform line with the source of harmonic voltage and the system in a steady state. This three-phase circuit can be replaced (as it is known from e.g.[1]) by an equivalent one-phase line, Fig. 1.

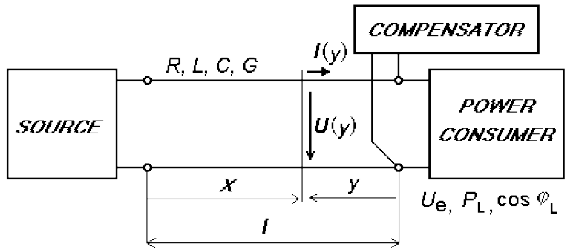

Fig. 1 Power supply and compensation circuit supply by a uniform line

If we label the parameters of the equivalent one-phase homogenous line for a length unit *R, L, C, G*, the course of voltage and current along this line is given by the known relations:

$$\boldsymbol{U}(y) = \boldsymbol{U}_e \cosh(\boldsymbol{\gamma} y) + \boldsymbol{Z}_0 \boldsymbol{I}_e \sinh(\boldsymbol{\gamma} y)$$

$$\boldsymbol{I}(y) = \frac{\boldsymbol{U}_e}{\boldsymbol{Z}_0}\sinh(\boldsymbol{\gamma} y) + \boldsymbol{I}_e \cosh(\boldsymbol{\gamma} y) \tag{1}$$

where

$$\boldsymbol{Z}_0 = \sqrt{\frac{R + \mathrm{j}\omega L}{G + \mathrm{j}\omega C}},$$

$$\boldsymbol{\gamma} = \sqrt{(R + \mathrm{j}\omega L)(G + \mathrm{j}\omega C)} \tag{2}$$

represent the wave impedance and line propagation constant, and $\boldsymbol{U}_\mathrm{e}$, $\boldsymbol{I}_\mathrm{e}$ are phasors of voltage and current on the terminals of the load.

The complex power supplied to the load is:

$$\boldsymbol{S}_\mathrm{L} = \boldsymbol{U}_\mathrm{e} \boldsymbol{I}_\mathrm{e}^* = P_\mathrm{L} + \mathrm{j} Q_\mathrm{L} \tag{3}$$

where $\boldsymbol{I}_\mathrm{e}^*$ is a complex phasor that is conjugated with phasor $\boldsymbol{I}_\mathrm{e}$, P_L is the active input of the load, and Q_L is reactive input.

$$Q_\mathrm{L} = P_\mathrm{L}\, \mathrm{tg}\, \varphi_\mathrm{L} = P_\mathrm{L} \frac{\sqrt{1 - \cos^2 \varphi_\mathrm{L}}}{\cos \varphi_\mathrm{L}} \tag{4}$$

Compensation is accomplished through a compensation circuit consisting of a series connection of power factor capacitors of capacity C_c, and a coil of inductance L_c. The compensation circuit is connected to the terminals of the load. Resonance frequency of this series circuit is 190 Hz. Resonance frequency is identified so, in order to be away from frequency higher harmonic in network. For the concrete *Cc* with using the relation

$$\omega_\mathrm{r} = \frac{1}{\sqrt{L_\mathrm{c}\, C_\mathrm{c}}} \tag{5}$$

we calculate *Lc*.

The complex output that is delivered to the compensation circuit is described as follows:

$$\boldsymbol{S}_\mathrm{c} = \boldsymbol{U}_\mathrm{e} \boldsymbol{I}_\mathrm{c}^* = \boldsymbol{U}_\mathrm{e}\, \mathrm{j} \frac{\boldsymbol{U}_\mathrm{e}^*}{\omega L_\mathrm{c} - \dfrac{1}{\omega C_\mathrm{c}}} \tag{6}$$

After substituting for L_c from eq. (5) we obtain:

$$\boldsymbol{S}_c = \mathrm{j}\,\frac{\omega\omega_r^2 C_c \boldsymbol{U}_e \boldsymbol{U}_e^*}{\omega^2 - \omega_r^2} \tag{7}$$

At the end of the line the complex power is:

$$\begin{aligned}\boldsymbol{S} = \boldsymbol{S}_L + \boldsymbol{S}_C &= \\ &= P_L + \mathrm{j}\left(Q_L + \frac{\omega\,\omega_r\,C_c\,\boldsymbol{U}_e\,\boldsymbol{U}_e^*}{\omega^2 - \omega_r^2}\right)\end{aligned} \tag{8}$$

Voltage and current are distributed along the line according the relations:

$$\boldsymbol{U}(y) = \boldsymbol{U}_e \cosh(\gamma y) + \boldsymbol{Z}_0 \left(\frac{\boldsymbol{S}}{\boldsymbol{U}_e}\right)^* \sinh(\gamma y) \tag{9}$$

$$\boldsymbol{I}(y) = \frac{\boldsymbol{U}_e}{\boldsymbol{Z}_0} \sinh(\gamma y) + \left(\frac{\boldsymbol{S}}{\boldsymbol{U}_e}\right)^* \cosh(\gamma y) \tag{10}$$

The following relation describes active losses in the line:

$$\begin{aligned}\Delta P = R \int_0^l \boldsymbol{I}(y)\,\boldsymbol{I}^*(y)\,\mathrm{d}y \; + \\ + \; G \int_0^l \boldsymbol{U}(y)\boldsymbol{U}^*(y)\,\mathrm{d}y\end{aligned} \tag{11}$$

or more simply

$$\Delta P = P(l) - P_L \tag{12}$$

where $P(l)$ is active output delivered by the source.

The optimisation task is formed as follows: we know $\boldsymbol{U}_e$, P_L, $\cos\varphi_L$, R, L, C, G (or $\boldsymbol{Z}_0$, γ), l. We search the value C for which the active losses are minimal,

$$\Delta P = \min. \tag{13}$$

As it is a one-parameter optimisation task, its numerical solution is easy. To be able to investigate the minimum neighbourhood, we design the course of dependence $\Delta P = f(C_c)$ and find the co-ordinates of the global minimum (ΔP_{min}; $C_{c\,min}$).

2.2 Non-linear load

The load is considered to be a converter working to the load that has an inductive character. The waveform of the current is (Fig. 2).

$$\begin{aligned} i_z(\omega t) = 0,\ &\omega t \in (0, \varphi) \\ &\omega t \in (\varphi + d, \varphi + d + k) \\ &\omega t \in (\varphi + 2d + k, \pi)\end{aligned} \tag{14}$$

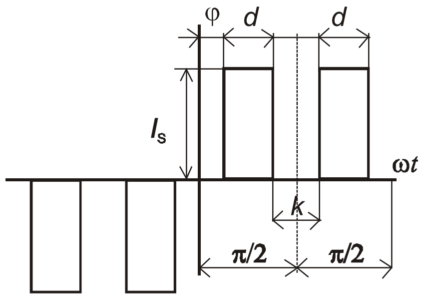

Fig. 2 Course of the current drawn by the load

$$\begin{aligned} i_z(\omega t) = I_S,\ &\omega t \in (\varphi, \varphi + d) \\ &\omega t \in (\varphi + d + k, \varphi + 2d + k) \\ &\omega t \in (\varphi + 2d + k, \pi)\end{aligned} \tag{15}$$

It is an odd function and thus only the sinus function can be used with the amplitude:

$$\begin{aligned} b_n &= \frac{2}{\pi}\int_0^\pi i_z(\omega t)\sin(n\omega t)\,\mathrm{d}(\omega t) = \\ &-\frac{4 I_S}{n\pi}\left[\sin(nk/2) - \sin(nk/2 + d)\right]\sin(nd/2)\end{aligned} \tag{16}$$

If we consider a symmetrical network at 3-phase bridge converter, then $d + k = 60^{\mathrm{O}}$. Therefore we obtain that:

$$b_n = \frac{8 I_S}{n\pi}\sin(nd/2)\cos(n\pi/2)\sin(n\pi/2) \tag{17}$$

For higher order harmonics, we can then write:

$$I_n = \frac{\sin(nd/2)}{n\sin(d/2)} I_1 \tag{18}$$

where

$$\begin{aligned} I_1 &= \frac{8 I_S}{\pi}\sin(d/2)\cos(\pi/6) = \\ &= 2.205\, I_S \sin(d/2)\end{aligned} \tag{19}$$

After substituting for $2d = 2\pi/3$ we obtain the relation:

$$I_n = \frac{I_1}{n} \tag{20}$$

The phase shift between harmonics is given by:

$$\varphi_n = \arctan(a_n / b_n) \tag{21}$$

It is an odd function. For an odd function it is valid that Fourier series contains only sinus members. From formula (21) it is evident that the phase shift of harmonics is zero.

Limited to the first three harmonics, the current taken by the load is:

$$i_S(0,t) = \sqrt{2}I_1\sin(\omega t) + \sqrt{2}\frac{I_1}{3}\sin(3\omega t) + \sqrt{2}\frac{I_1}{5}\sin(5\omega t) \tag{22}$$

Current and voltage of a particular harmonic is calculated from equations:

$$\mathbf{U}(\mathrm{x}) = \mathbf{A}e^{\gamma x} + \mathbf{B}e^{-\gamma x} \tag{23}$$

$$\mathbf{I}(\mathrm{x}) = -\frac{\mathbf{A}}{\mathbf{Z_o}}e^{\gamma x} + \frac{\mathbf{B}}{\mathbf{Z_o}}e^{-\gamma x} \tag{24}$$

The known boundary conditions are:

$$\boldsymbol{U}(0) = \boldsymbol{U}_\mathrm{p},\ \boldsymbol{I}(l) = \boldsymbol{I}_\mathrm{k} \tag{25}$$

After substituting the conditions (25) into the equation (23 and 24) we obtain:

$$\boldsymbol{U}_\mathrm{p} = \boldsymbol{A} + \boldsymbol{B} \tag{26}$$

$$\mathbf{I}_\mathrm{k} = -\frac{\mathbf{A}}{\mathbf{Z_o}}e^{\gamma l} + \frac{\mathbf{B}}{\mathbf{Z_o}}e^{-\gamma l} \tag{27}$$

In solving this equation, the coefficients of $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined, giving us:

$$\mathbf{U}(x) = \frac{\sinh(\gamma x)}{\cosh(\gamma x)}(\mathbf{U}_\mathrm{p}e^{-\gamma l} - \mathbf{Z}_0\mathbf{I}_\mathrm{k}) + \mathbf{U}_\mathrm{p}e^{-\gamma x} \tag{28}$$

In substitution for

$$\mathbf{I}_\mathrm{k} = \frac{\mathbf{U}_\mathrm{k}}{\boldsymbol{Z}_\mathrm{LC}} + \mathbf{I}_\mathrm{S} \tag{29}$$

where $\boldsymbol{Z}_\mathrm{LC}$ is the compensator impedance and $\boldsymbol{I}_\mathrm{S}$ is the current produced by the converter.

We get the relation for the voltage at the end of the line:

$$\mathbf{U}_\mathrm{k} = \frac{\frac{\sinh(\gamma l)}{\cosh(\gamma l)}(\mathbf{U}_\mathrm{p}e^{-\gamma l} + \mathbf{Z}_0\mathbf{I}_\mathrm{S}) + \mathbf{U}_\mathrm{p}e^{-\gamma l}}{1 + \frac{\mathbf{Z}_0}{\mathbf{Z}_\mathrm{LC}}\frac{\sinh(\gamma l)}{\cosh(\gamma l)}} \tag{30}$$

From formula (29) it is possible to define the voltage of particular harmonics at the end of the line and then current at the end of the line $\boldsymbol{I}_\mathrm{k}$. Providing the source does not produce any voltage that contains harmonics of higher orders, then in formula (30) the voltage at the beginning of the line of each harmonic equals zero. Then it is possible to write down this relation for the harmonic of order n in the form

$$\mathbf{U}_{\mathrm{k}(n)} = \frac{\frac{\sinh(\gamma_n l)}{\cosh(\gamma_n l)}(\mathbf{Z}_0\mathbf{I}_{\mathrm{S}(n)})}{1 + \frac{\mathbf{Z}_{0(n)}}{\mathbf{Z}_{\mathrm{LC}(n)}}\frac{\sinh(\gamma_{(n)} l)}{\cosh(\gamma_{(n)} l)}} \tag{31}$$

Losses in homogenous lines are calculated as follows: we define instantaneous value of current courses in particular harmonics in homogenous line. Then we add these currents and calculate the final loss from it

$$\Delta P = \frac{R}{T}\int_0^T\int_0^l i^2(y,t)\,\mathrm{d}t\,\mathrm{d}l$$

(when neglecting the losses in insulation)

3. SIMULATIONS RESULTS

3.1 Linear load

Input values of l*oad*: U_e = 110 kV, P_L = 21,45 MW, cos φ_L = 0,65, f = 50 Hz. Cable lines have parameters (according to standard ČSN 332000-5-523): R = 0,153 Ω/km, L = 1,238 mH/km, C = 10 nF/km, G = 0,1 μS/km and its length is l = 500 km.

a) *Compensation by condenser C_c only* ($L_c = 0$)
Dependence of losses on the lines in capacity C_c has its minimum for $C_{\mathrm{c\ min}}$ = 4,3 μF. Now the losses in lines reach the value ΔP_{min} = 3,7025 MW. In Fig. 4 the courses of voltage and current along the lines in this optimal compensation are shown.

b) *Compensation by series circuit L_c and C_c*
A graph of function $\Delta P = \mathrm{f}(C_\mathrm{c})$ is shown in Fig. 5. Co-ordinates of minimum are: ΔP_{min} = 3,7026 MW, $C_{\mathrm{c\ min}}$ = 4 μF. Fig. 6 shows the courses of voltage $U(y)$ and current $I(y)$ along the lines for compensation circuit LC, where C_c = 4 μF, L_c=175mH.

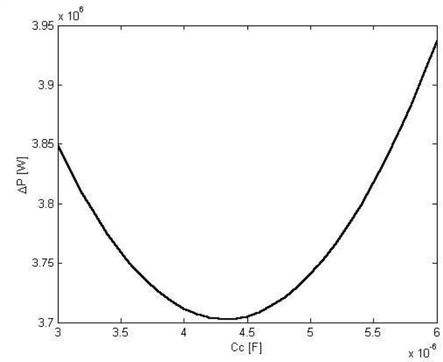

Fig. 3 Dependence of losses ΔP in the lines on capacity C_c. Compensation by capacitor only

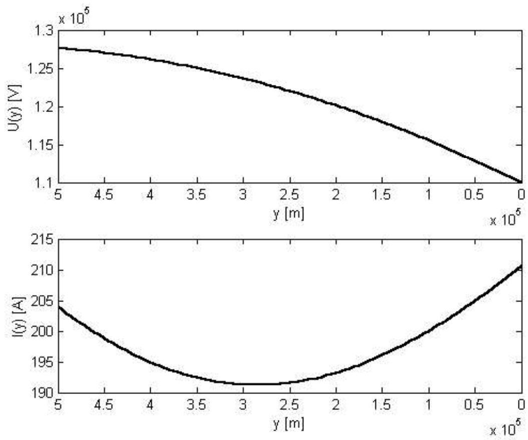

Fig. 4 Course of voltage U(y) and current I(y) along the lines at optimal compensation

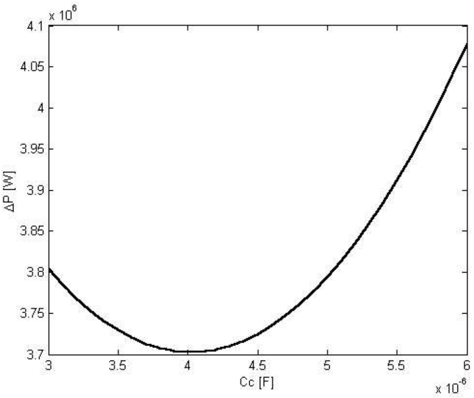

Fig. 5 Dependence of losses in the lines ΔP on capacity C_c . Compensation circuit L_c $\boldsymbol{C_c}$

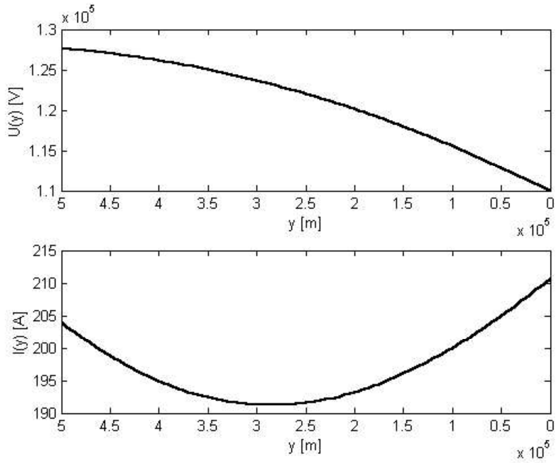

Fig. 6 Course of voltage $U(y)$ and current $I(y)$ along the lines at $C_c = 4$ μF. Compensation circuit contain L_c and C_c

3.2 Non-linear load

For non-linear loads in the course of losses on capacity C_c, two local minima were found. Phase shift of current of higher harmonics was considered the same. The compensator consists of a series connection L_c, C_c, with a resonance frequency of 190 Hz. The course of losses along the line is in dependence on capacity (Fig. 7). The course of losses has the minimum for $C_{c\ min} = 1{,}9$ μF, $\Delta P_{min} = 6{,}4$ MW. Simulation was carried out by MATLAB

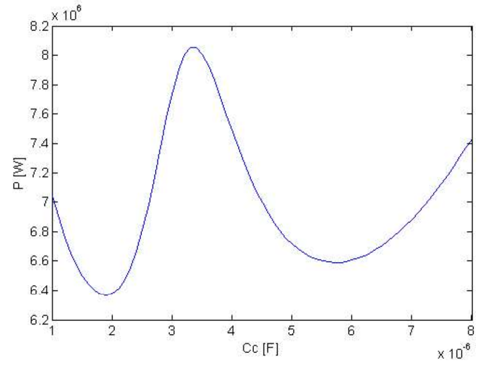

Fig. 7 Dependence of losses ΔP along the line on capacity C_c . Compensation circuit L_c , C_c that loads the current contains harmonics of order 1, 3, 5

programme [3]. It is evident that calculation based on distributed parameters of line increases efficiency of compensation and led to decreased losses in the line.

4. CONCLUSION

The method was described of calculating compensation circuit parameters L_c a C_c for a non-linear load, which is supplied from a uniform line. This method can be used for solving more general optimisation problems of the design of compensation circuit L_c and C_c for a non-linear load. From this article it is evident that methods currently used in practice for the compensation circuit design are not fully optimal when it comes to minimising losses in the line.

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BIOGRAPHY

Daniel Mayer was born in Pilsen (Czech Republic) on August 8, 1930. He received the Ing., PhD. and DSc degrees in electrical engineering from Technical University of Prague (Czech Republic) in 1952, 1958 and 1979, respectively. In 1956 he began his professional career as a Senior Lecturer and later as a Associate Professor at the University of West Bohemia in Pilsen. In 1968 he was appointed Full Professor of the Theory of Electrical Engineering. Many years he has been head of the Institution of Theory of Electrical Engineering. His main teaching and research interests include circuit theory, electromagnetic field theory, electrical machines and history of electrical engineering. He has published 6 books and over 200 scientific papers.

Jan Hanák was born on December 26, 1976. In 2001 he graduated from the University of West Bohemia. He is currently working as a doctoral student at University of West Bohemia in Pilsen, in the Department of Theory of Electrical Engineering. His research includes electromagnetic field theory and quality of transmission of electromagnetic energy in distribution line.