A NEW APPROACH TO POWER SYSTEM TRANSIENT STABILITY ASSESSMENT

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SUMMARY

In this article is presented a simple method for transient stability assessment of a single machine-infinite bus power system using catastrophe theory. Catastrophe theory is a mathematical technique for the qualitative analysis of system equations, defining the jump phenomena and sudden changes caused by smooth alterations in the situation.

The computation of critical fault clearing time and critical fault clearing angle has been demonstrated and compared with results obtained by the numerical step-by-step method. The technique is performed in such a way that individual machine energies are balanced. It is valid for any fault type or location and is effective for on-line stability assessment.

Keywords: *transient stability, power system, catastrophe theory, critical fault clearing time, critical fault clearing angle*

1. INDRODUCTION

Transient stability in a power system can be defined as the ability to retain synchronism with the system, following a large disturbance such as a sudden, large change in load, or a generation and transmission system fault which has occurred and then been removed. An understanding of system stability requires a knowledge both of the mathematical modelling of the problems and of numerical techniques. However, the computing of power system stability is extremely intricate and highly nonlinear problem.

2. TECHNIQUES FOR TRANSIENT STABILITY ASSESSMENT

Transient stability has always been studied in practice via numerical integration of differential equations for a given fault and examining the simulated system response. The research is concerned to the real-time or faster than real-time power system transient stability simulations.

The numerical methods allow accurate and detailed representation of power system, but they are not suitable for on-line application due to large computation time. An alternative approach by the application of Lyapunov theory was proposed in 1966 by Gless, EL-Abiad and Nagapan [1]. Much work has been done to find ways and means to reduce the computation time for transient stability studies, particularly by the Lyapunov's method and pattern recognition method (both come under direct methods). In Lyapunov`s method, research has concetrated on finding the best Lyapunov function. The results obtained are still very conservative because Lyapunov`s method defines stability regions by means of state variables and the state variables must be updated for changes in operating conditions and fault locations. Pattern recognition has also been proposed to solve the transient stability directly and can be used for on-line applications. The main drawback to the method is in the excessive off-line computation that has to be done first. [2].

Many studies have been done using the concept of potential and kinetic energies to analyse the power system transient stability. This concept is based on the deduction of an energy function dependent on all system state variables. It has been applied to define the critical group of machine separated from the rest of the system to determine the overall system stability, but it needs further investigation.

Another applications still used to determine the critical transient energy and to identify the controlling unstable equilibrium point for a specific disturbance. These applications still use computational techniques that make it difficult for them to be used for on-line stability assessment. [2].

On the other hand, the catastrophe theory as a qualitative, effective technique has been applied to evaluate the power system transient stability.

3. CATASTROPHE THEORY

The catastrophe theory (CT) is the invention of Rene Thom, a French mathematician. CT provides certain mathematical models to demonstrate the number of qualitatively different configurations of discontinuities that can occur, which depend upon the control variables which are normally a few, and not upon the number of state variables which may be many. For system with not more than five control variables and two state variables, there are seven distinct forms of elementary catastrophes.

Assuming a family of function:

$$V : F \times C \to R \tag{1}$$

where: F - is a manifold, R^n, called state space,
C - another manifold, R^r, called control space.

A manifold is an indicative of a higher-order surface, e.g. one-dimensional manifold is a curve, a second-order manifold is a two-dimensional surface a third-order manifold is a three-dimensional surface. The catastrophe manifold N, is the subset $R^n \times R^r$ defined by:

$$\nabla_x . V_c(x) = 0 \tag{2}$$

where: $V_c(x) = V(x, c)$ - is the set of all critical points of all potentials V_C in the family V,
∇_x - is the partial derivative with respect to x.

Next, we find the singularity set S, which is the subset of the manifold N, that consist of all singular points of V. These are the points at which:

$$\nabla_x . V_c(x) = 0 \text{ and } \nabla_x^2 V_c(x) = 0 \tag{3}$$

The singularity set S is then projected down onto the control space R^r to obtain bifurcation set B. The bifurcation set is the image of catastrophe manifold N in the control space C. The bifurcation set B provide the projection of the stability region of all possible stable points of V in terms of the control variables, which usually represents the system parameters.

The seven elementary catastrophes for the control space dimension $r \leq 4$ have been well defined and explained [3]. From this seven elementary catastrophes is swallowtail catastrophe used in this paper.

4. APPLICATION OF CATASTROPHE THEORY TO TRANSIENT STABILITY PROBLEM

Wvong a Mihirig, 1985 first applied CT for transient stability analysis of one machine infinite bus power system using the swallowtail catastrophe model. Later on 1989, they used the cusp catastrophe model to evaluate the transient stability of multimachine power systems and compared results of critical fault clearing time with those obtained by the standard step-by-step method. Subsequently, Sallam, 1989 used energy balance approach for multimachine power system and used the swallowtail catastrophe for computation of the critical time and critical fault clearing angle [4].

The present research work applied differs significantly from the techniques described by Wvong and Mihirig, 1985, 1989, and Sallam, 1989. In this article, the computation of control parameters for one machine infinite bus power system is based on the exact initial machine angle of the power system which gives a more accurate estimate than the above mentioned computation techniques.

Consider an one machine infinite bus system as shown in Fig.1. Assume the three-phase fault on infinite bus.

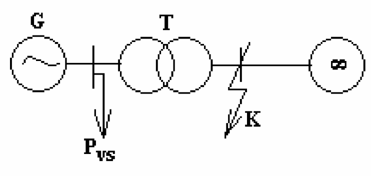

Fig. 1 One machine infinite bus power system

Data for system on Fig. 1 show Tab. 1.

Generator	Transformer	Load
P_{nG} = 220 MW S_{nG} = 259 MVA U_n = 15,75 kV $\cos\varphi$ = 0,85 x'_d = 26,7 % T_a = 2.H = 9,89 s	S_{nT} = 250 MVA U_{nT} = 15,75/420 kV u_k = 13,3 %	P_{vs} = 0,8 P_{nG}

Tab. 1 Data for system on Fig. 1. undertake from Slovak nuclear power plant Jaslovské Bohunice.

The swing equation of the system is given by:

$$M \frac{d^2\delta}{dt^2} = P_i - P_{\max} \sin\delta \tag{4}$$

where: M - inertia constant,
P_i - input power,
$P_{\max} \sin\delta$ - output power.

For a three-phase fault at location K, Fig.1. the system is stable only if the kinetic energy generated during the fault (E_k) is less than or equal to the potential energy during the post fault period (E_p). Consider the critical clearing case where:

$$E_k + E_p = 0 \tag{5}$$

Equation (5) can be implemented in terms of CT as the equilibrium surface or the manifold N of a smooth function V, i.e. we consider:

$$N = \nabla_x . V_C(x) = E_k + E_p = 0 \tag{6}$$

We also define the singularity set S, as the set of steady-state stability limits where:

$$\nabla_x^2 V_C(x) = 0 \qquad (7)$$

Eqn. (5) can be derived by integrating the swing equation to obtain:

$$\frac{1}{2} M \dot{\delta}_c^2 - P_{mA} \cos\delta_C - P_i\delta_C + P_m\delta_m + P_{mA}\cos\delta_m = 0 \qquad (8)$$

where: P_{mA} - is the maximum power after fault clearing,
δ_C - is the critical clearing angle,
$\dot{\delta}_C$ - is speed at critical clearing,
δ_m - is the unstable equilibrium angle (maximum angle).

Eqn. (5) represents the equilibrium surface of the system for all possible fault locations.
When we consider:

$$\dot{\delta}_c = \omega_C = \gamma . t_C \qquad (9)$$

Then:

$$\delta_C = \delta_0 + \frac{1}{2}\gamma . t_C^2 \qquad (10)$$

where: γ - is the acceleration in the instant of fault occurrence,
t_C - critical clearing time.

$$\text{Let: } x = \frac{1}{2}\gamma . t_C^2 \text{ and } K = P_i\delta_m + P_{mA}\cos\delta_m \qquad (11)$$

Next we must replacing goniometric functions by Taylor's series expansion. But we must know, how many coefficients in this series we can removed. This problem is called Taylor determinacy. Coefficients from cosine series expansion we can use just to the fourth order [3]:

$$M\gamma x - P_{mA}\left[\cos\delta_0 \cos x - \sin\delta_0 \sin x\right] - P_i\left(\delta_0 + x\right) + K = 0 \qquad (12)$$

By truncating the series expansion and taking the terms up to the fourth order, Eqn. (12) we can written:

$$-B_4x^4 - B_3x^3 + B_2x^2 + B_1x + B_0 = 0 \qquad (13)$$

where:

$$B_4 = \frac{P_{mA}}{576}\left(\delta_0^4 - 12\delta_0^2 + 24\right),$$

$$B_3 = \frac{P_{mA}}{36}\left(6\delta_0 - \delta_0^3\right)$$

$$B_2 = \frac{P_{mA}}{48}\left(\delta_0^4 - 12\delta_0^2 + 24\right), \qquad (14)$$

$$B_1 = M\gamma - \frac{P_{mA}}{6}\delta_0^3 + P_{mA}\delta_0 - P_i,$$

$$B_0 = \frac{P_{mA}}{2}\delta_0^2 - \frac{P_{mA}}{24}\delta_0 - P_i\delta_0 - P_{mA} + K$$

Next we eliminate the x^3 term by taking:

$$x = y - \alpha \text{ and } \alpha = \frac{B_3}{4B_4} \qquad (15)$$

Eqn. (13) we can written in the form of swallowtail catastrophe manifold:

$$\nabla_y V(y,c) = y^4 + uy^2 + vy + w = 0 \qquad (16)$$

where: u , v and w - are the control variables:

$$u = -\frac{1}{B_4}\left(B_2 + \frac{3}{8}\frac{B_3^2}{B_4}\right)$$

$$v = -\frac{1}{B_4}\left(B_1 - \frac{B_2}{2}\frac{B_3}{B_4} - \frac{1}{8}\frac{B_3^3}{B_4^2}\right) \qquad (17)$$

$$w = -\frac{B_0}{B_4} + \frac{B_1}{4}\frac{B_3}{B_4^2} - \frac{B_2}{16}\frac{B_3^2}{B_4^3} - \frac{3}{256}\frac{B_3^4}{B_4^4}$$

The singularity set S can be obtained by differentiating the Eqn. (16):

$$\nabla_y^2 V(y,c) = 4y^3 + 2uy + v = 0 \qquad (18)$$

Eqn (16) and (18) can be used to find the bifurcation set B (Fig. 2). For the transient stability, it is necessary that all the points must lie in the bifurcation set as well as it must be greater than α .

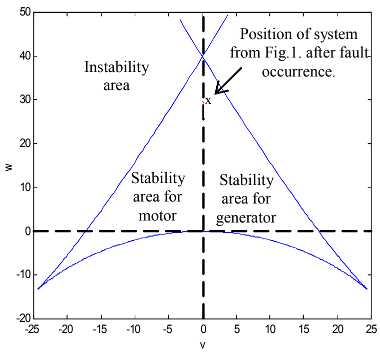

Fig. 2 Bifurcation set B which represents stability region

Physically the smallest positive root, which satisfies the inequality condition $y > \alpha$ represents the critical clearing time. Eqn. (16) has real roots in the bifurcation set region: two positive and two negative. The function V and V' can then sketched as shown in Fig. 3. The first root of $V(y)$ represents the critical clearing time.

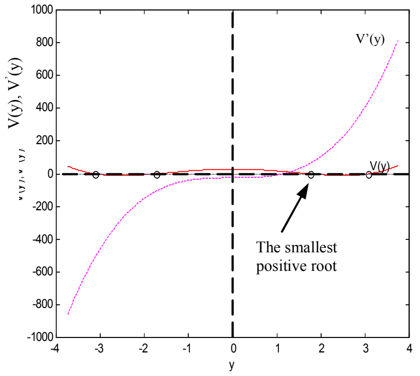

Fig. 2 V and V' for $v = 0{,}2672$ and $w = 29{,}2380$

The critical clearing time can be computed from the know value of δ_c, or $(y-\alpha)$, or x:

$$t_C = \sqrt{\frac{2(y-\alpha)}{\gamma}} \tag{19}$$

Practical results from this theory are shown in Tab.2. For the one machine infinite bus system on Fig. 1. we obtain critical clearing time and critical clearing angle values. These results are compared wits results obtained by the classical step-by-step numerical method as well as by Wvong and Mihirg approach.

Variants	*Critical clearing time* t_C [s]		
	0,8.S$_n$	***S$_n$***	***1,2.S$_n$***
Step-by-step	0,3332	0,3242	0,3151
Variant A	0,3390	0,3290	0,3190
Variant B	0,3374	0,3274	0,3175

Variants	*Critical clearing angle* δ_C [°]		
	0,8.S$_n$	***S$_n$***	***1,2.S$_n$***
Step-by-step	102,3948	99,1871	96,0861
Variant A	105,3847	101,5609	97,9916
Variant B	104,5137	100,7654	97,2616

Tab. 2 Comparison of critical clearing angle and critical clearing time computing by various techniques for generator on Fig. 1:
Variant A – Wvong and Mihirig approach
Variant B – proposed method

5. CONCLUSION

Catastrophe theory has been applied to the transient stability assessment of one machine infinite bus system. The advantages of the proposed method are:

a) Transient stability region is obtained in terms of a small number of so-called control variables instead of a usually large number of state variables. The control variables are functions of the system parameters and state variables.
b) The stability region is valid for any loading condition or fault location. Degree of stability is indicated by the distance of operating point from boundaries of the stability region.
c) The proposed method is easy and provides good agreement with standard numerical step-by-step methods.

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BIOGRAPHY

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