

SPURIOUS IN PLL-DDS FREQUENCY SYNTHESIZERS

*Věnceslav F. KROUPA, *Jarmil ŠTURSA

*ÚŘE AV ČR, Chaberská 57, 182 51 Praha 8,

kroupa@ure.cas.cz, stursa@ure.cas.cz, Tel.: +420 2 688 1804, Fax: +420 2 688 0222

SUMMARY

We will discuss the basic parameters of the steering signal that is built in the S-D modulator and that controls pulse removing in the PLL frequency synthesizer. The knowledge of these parameters is important for the choice of the bias of the steering signal and for the correct evaluation of the synthesizer spectrum. We shall investigate the dependence of these parameters on the order of the S-D modulator and show the spurious phase spectrum of the considered fractional-N synthesizer for the third and fourth modulator order.

Keywords: frequency synthesizers, phase locked loops, digital signal processing

1. INTRODUCTION

Modern communications require synthesizers with narrow channels, high density of channels, and rapid switching between them. The combination of phase locked loops (PLL) [1] and direct digital frequency synthesizers (DDS) meets all these properties. To arrive at the narrow channels the fractional-N frequency synthesizer were introduced and means for reduction of produced spurious signals, with the assistance of Σ - Δ modulators, were suggested [2]. Recently these systems have found practical applications in commercial communications. They are suitable in microwave frequency region and in mobile telecommunications [3,4].

The Σ - Δ modulator is composed from a chain of m digital accumulators of the size Y ($Y=2^R$, R is the accumulator word length) and of the shaping network processing the overflow signals of these accumulators. We can imagine these overflow signals as the output signals of a 1-bit quantizer.

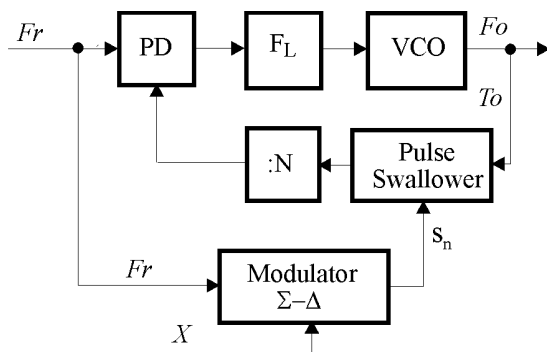


Fig. 1 Block diagram of the fractional-N frequency synthesizer

The principal idea of the operation of fractional-N frequency synthesizers is based on quasiperiodic removing (swallowing) of pulses from the VCO pulse train according the value of the steering signal

s_n (output signal of the shaping network) as illustrated in Fig. 1. The mean value of this steering signal is equal to the relation $X/Y=\xi$.

The modulator is operating with the clock frequency equal to reference frequency F_r . The fractional part of the division ratio (of the desired frequency) of the synthesizer can be controlled by the digital word X driving the first accumulator of the modulator.

2. OPERATION OF THE Σ - Δ MODULATOR

We shall suppose identical operating word length R_i ($R_i=R$) of all accumulators and their size $Y_i=Y$, where

$$Y_i = 2^{R_i} \quad (1)$$

The authors [5] consider Σ - Δ modulator of the 3rd order ($m=3$). Its block diagram is illustrated in Fig. 2.

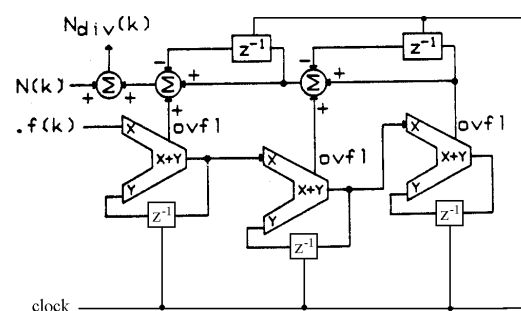
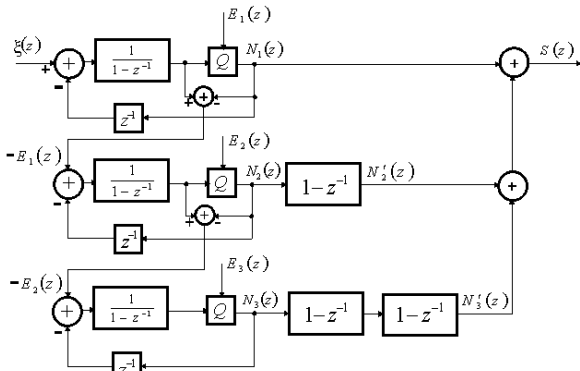


Fig. 2 Adapted Σ - Δ modulator of the third order

We shall explain the operation of this modulator more deeply on the model in Fig. 3, where we use the Z-transform of signals and transfer functions of the involved blocks. Each accumulator produces two output signals. The first one (E_i , $E_i < 1$), is given by the digital accumulation (integration) of the constant input signal ξ . The "overflow" signal N_i is the second signal and it acquires value $N_i=1$ in the case

of accumulator overflow and value $N_i=0$ in the other moments. The delay block z^{-1} in the feedback path models the addition modulo 1 of the accumulator.



This procedure is the source of overflowing. We can imagine N_i as the output signal of the 1-bit quantizer represented by the block Q of unity gain and by quantization error E_i . The signals $-E_1$ and $-E_2$, are input signals of the subsequent accumulators.

Fig. 3 The 3rd order modulator model

The overflow signal of the first accumulator can be expressed as

$$N_1(z) = (\xi(z) - N_1(z) \cdot z^{-1}) \cdot \frac{1}{1-z^{-1}} - E_1(z) \quad (2)$$

and after simple modification as

$$N_1(z) = (\xi(z) + E_1(z)) \cdot (1-z^{-1}) \quad (3)$$

The other overflow signals have the analogical form:

$$N_2(z) = (-E_1(z) + E_2(z)) \cdot (1-z^{-1}) \quad (4)$$

$$N_3(z) = (-E_2(z) + E_3(z)) \cdot (1-z^{-1}) \quad (5)$$

The overflow signal $N_i(z)$ is multiplied in the shaping network and the results are summed. The contribution of the first accumulator is eliminated by addition the contribution of the second accumulator etc. The resulting sum represents the Z-transform $S(z)$ of the steering signal s_n

$$S(z) = \xi(z) + (1-z^{-1})^3 \cdot E_3(z) \quad (6)$$

The variance of $E_{3,n}$ corresponds to

$$\text{var}(E_{3,n}) = \frac{1}{12} \quad (7)$$

The ideal mean value $M(E_{3,n})$ of one spectral line is equal to $\text{var}(E_{3,n})$ divided by Y

$$M(E_{3,n}) = \frac{1}{12 \cdot Y} \quad (8)$$

3. INVESTIGATED PARAMETERS

We have simulated the Σ - Δ modulator of different order m ($m=1..4$) The simulation confirmed that the steering signal is a periodical integer-value signal, random inside its period. It has the normal-like distribution and its variance depends strongly on the modulator order m and weakly on the accumulator width R . Its repetition period T_s and the

range Δ_s of possible amplitude values are functions of the modulator order (Tab. 1). The period T_s is here expressed in multiples of Y , the range Δ_s is given by the extreme integer values of s_n .

Table 1 Period and range of the steering signal

Order m	Period T_s	Range Δ_s	Number of levels
1	1Y	< 0, 1>	2
2	2Y	< -1, 2>	4
3	2Y	< -3, 4>	8
4	4Y	< -7, 8>	16

The period T_s was determined by the autocorrelation function during the simulation of the Σ - Δ modulator. Its knowledge is important for computation of the spectrum of the fractional-N synthesizer.

The range Δ_s was adjusted by considering all possible values of overflow signals $N_{i,n}$ of the three accumulators. Its knowledge is important for biasing the swallowing process, since the steering signal is bipolar and only positive number of pulses can be removed in the pulse swallower.

We have also investigated the variances of the time error signal $E_{3,n}$, of the overflow signals $N_{i,n}$ and of the steering signal s_n .

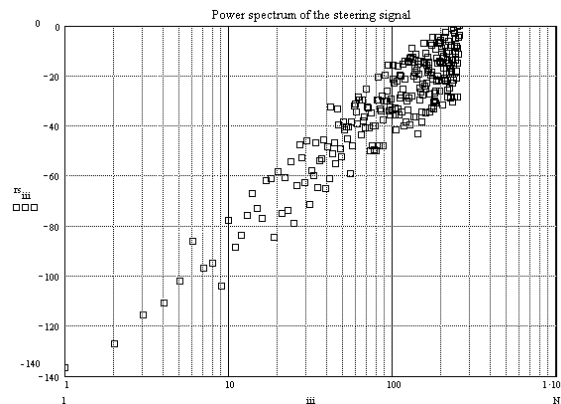


Fig. 4 Power spectrum of the steering signal

The variance of $E_{3,n}$ takes the value $\sigma_3 \approx 0.083 \approx 1/12$ (for $R>3$) that corresponds to the random signal with uniform probability distribution.

The overflow signals $N_{i,n}$ take only two values (0, 1) with practically the same probability 0.5, and have for $i>1$, the variance $\sigma^2=0.25$. Practically the same results we got for $m=2,3,4$.

The power spectrum of the steering signal computed with the help of FFT is for system parameters $m=3$, $X=13$, $R=8$ and $H=2$ in Fig. 4. We can see here the low level of spurious frequency components in the nearness of the carrier frequency ($iii=1$) and their increasing with the slope 60 dB/decade.

4. EFFECTIVE DIVISION RATIO

The output synthesizer frequency F_o is associated in the locked state of the phase loop with the reference frequency F_r by

$$F_o = N_{eff} \cdot F_r \tag{9}$$

or in the time domain the repetition period T_o of the output signal is given by

$$T_o = \frac{T_r}{N_{eff}} \tag{10}$$

where $T_r=1/F_r$ and where N_{eff} is the effective division ratio in the system

The pulse sequence on the feedback port of the phase detector is affected by the operation of the divider N , by the pulse swallower and by the sampling moments of the synthesizer output signal. The “periods” of this signal are, therefore, time varying. The pulse swallowing and time quantization produce the phase modulation of the processed signal and changes the division ratio to the effective value N_{eff} which consist of three parts:

$$N_{eff} = N + \xi(z) + \Delta N_{eff} \tag{11}$$

where N is integer part of the division ratio, $\xi(z)$ is fractional part of the division ratio and ΔN_{eff} is noise contribution of the Σ - Δ modulator.

This third term we must take into account in investigating the synthesizer output spectrum. Due to the phase modulation of the feedback signal we get the following expression of the synthesizer output frequency

$$F_o(z) = (N + \xi(z)) \cdot F_r + (1 - z^{-1})^3 \cdot E_3(z) \cdot F_r \tag{12}$$

The first term represents the desired frequency (N and $\xi(z)$ are constant for multiples of the period T_s), the second one represents the noise contribution due to fractional division. The value of this contribution ΔN_{eff} depends on the order m of the Σ - Δ modulator

$$(1 - z^{-1})^m \tag{13}$$

and on the value of $E_{3,m}$ i.e. on the quantization error of the last accumulator.

After integration we arrive at the envelope of the power phase spurious signals for the modulator of the 3rd order ($m=3$)

$$L(n) = \frac{(2 \cdot \pi)^2}{12 \cdot Y} \left[2 \cdot \sin\left(\pi \cdot n \cdot \frac{F_r}{2 \cdot Y}\right) \right]^{2(3-1)} \tag{14}$$

and for general m

$$L(n) = \frac{(2 \cdot \pi)^2}{12 \cdot Y} \left[2 \cdot \sin\left(\pi \cdot n \cdot \frac{F_r}{2 \cdot Y}\right) \right]^{2(m-1)} \tag{15}$$

where $n=1,2,\dots$

5. PHASE SPECTRUMS

We shall investigate the power spectrum of the synthesizer output signal and its noise properties. For this investigation we use programming possibilities of Mathcad and assume that the

synthesizer is operating under real conditions with real component parameters. We assume further that spurious signals in the Σ - Δ frequency synthesizers exceed all the noises generated in the PLL loop (in dividers, phase detectors, amplifiers). Consequently we can reduce the relation for the synthesizer output noise PSD (power spectral density) [1] to

$$S_{\phi,out}(f) \approx [S_{\phi,ref}(f) \cdot N^2 + \sum_{n=0}^{\infty} S(n)] \cdot |H_{eff}(f)|^2 + S_{\phi,VCO}(f) \cdot |1 - H_{eff}(f)|^2 \tag{16}$$

where $S_{\phi,out}(f)$ is PSD of the output signal, $S_{\phi,ref}(f)$ is PSD of the reference signal, $H_{eff}(f)$ is transfer function of PLL, $S_{\phi,VCO}(f)$ is PSD of the VCO and $S(n)$ spectral components of the Σ - Δ modulator.

The power spectrum of individual quantities is shown in Fig. 5 and Fig. 6. Both figures hold for the PLL system of the 4th order, type 2. Their open loop transfer function is given by the expression

$$G5_m = G_m \cdot Ge_m \cdot g3_m \cdot g4_m \tag{17}$$

where G_m is the transfer function of the 2nd order

$$G_m = \frac{j \cdot x_m \cdot 2 \cdot \zeta + 1}{(j \cdot x_m)^2} \tag{18}$$

x_m is frequency normalized by the natural one, ζ is the damping factor and Ge_m is the transfer function of the phase detector

$$Ge_m = 1 \cdot e^{-j \cdot x_m \cdot \delta} \tag{19}$$

Factor δ is ratio of the natural frequency F_n to twice reference frequency F_r . In our case $\delta=0.000153$. Transfer functions $g3_m$ and $g4_m$ of filters are given as

$$g3_m = \frac{1}{1 + 2 \cdot j \cdot x_m \cdot \zeta \cdot \kappa} \tag{20}$$

$$g4_m = \frac{1}{1 + 2 \cdot j \cdot x_m \cdot \zeta \cdot \eta} \tag{21}$$

Following two figures hold for the same system parameters $R=9$, $N=140$, $N_{eff}=140.625$ and filter parameters $\zeta=0.7$, $\kappa=0.2$ and $\eta=0.03$, but for the different modulator order m . This order takes value $m=3$ in Fig. 5, and $m=4$ in Fig. 6.

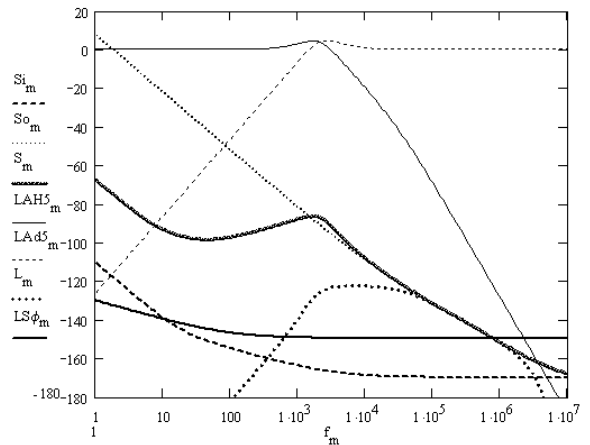


Fig. 5 PLL properties for $m=3$ and $R=9$, $N=140$, $N_{eff}=140.625$, $\zeta=0.7$, $\kappa=0.2$, $\eta=0.03$

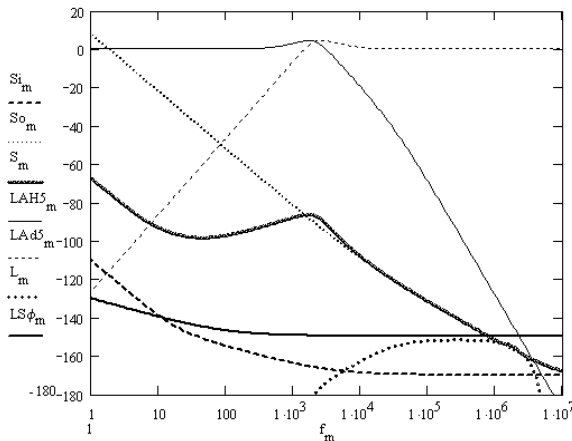


Fig. 6 PLL properties for $m=4$ and:
 $R=9$, $N=140$, $N_{eff}=140.625$, $\zeta=0.7$, $\kappa=0.2$, $\eta=0.03$

Symbols in these figures correspond to following meaning:

$$S_i \rightarrow 10 \cdot \log(S_{\varphi,ref})$$

$$S_o \rightarrow 10 \cdot \log(S_{\varphi,VCO})$$

$$S \rightarrow 10 \cdot \log(S_{\varphi,out})$$

$$LAH5 \rightarrow 10 \cdot \log(|H_{eff}|^2)$$

$$LAd5 \rightarrow 10 \cdot \log(|1 - H_{eff}|^2)$$

$LS\varphi$ → additive noise of PLL blocks

L → envelope of spurious signals due to modulator

6. CONCLUSIONS

We have shown the model of the Σ - Δ modulator and have given the important parameters of the steering signal for several values of the modulator order. Including the Σ - Δ modulator in the PLL makes it possible to achieve a fractional value of the synthesizer divider, but it produces spurious signals. The level of phase spurious spectral components is very small near the carrier and it is growing with the slope 40 dB/dec (for $m=3$) with the increasing distance from the carrier. We have also shown some results of the simulation of such frequency synthesizer operating with practical parameters values. This investigation demonstrates that the order $m=3$ of the Σ - Δ modulator is sufficient to obtain small spurious spectral components.

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BIOGRAPHY

Věnceslav F. Kroupa received the Ing. degree (M.Sc. diploma) in electrical engineering from Czech Technical University, and DrSc. degree from the Czechoslovak Academy of Sciences, Prague. In 1955 he joined the Institute of Radio Engineering and Electronics of the Czechoslovak Academy of Sciences, now Academy of Sciences of the Czech Republic with affiliation as a consultant. For the past 50 years his major scientific and research interests have been standard time and frequency, frequency stability and noise, frequency synthesis, precise frequency measurements, and phase locked loops. On these subjects he has written 5 books and published over 100 technical papers and reports. He holds 15 patents. Dr. Kroupa is Fellow IEEE, associated with the Ultrasonics, Ferroelectrics, and Frequency Control Society.

Jarmil Štursa finished his studies on the Faculty of electrical engineering of the Czech Technical University in 1956. Then he has been working in the Research Institute of Radio Engineering in Opočíněk in the field of radar receivers and since 1960 in Institute of Radio Engineering and Electronics of the Czechoslovak Academy of Sciences in Prague. After finishing doctoral studies (CSc. degree) in 1968 he has been working in the field of signal processing. He was interested particularly in the analysis of limited word length effects in digital filtering and in FFT. Since 1980 he has been working in the field of the speech signal analysis and synthesis. In last years he is interested in limited word length in the spectrum analysis algorithms, in the direct digital frequency synthesizers and in the simulation of these systems. Since 1994 he took part in three grant projects of the Grant agency of the Czech Republic.