FREQUENCY-DEPENDENT EXPRESSION FOR MUTUAL IMPEDANCE PER UNIT LENGTH OF INTERCONNECTS ON LOSSY SEMICONDUCTING SUBSTRATE

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SUMMARY

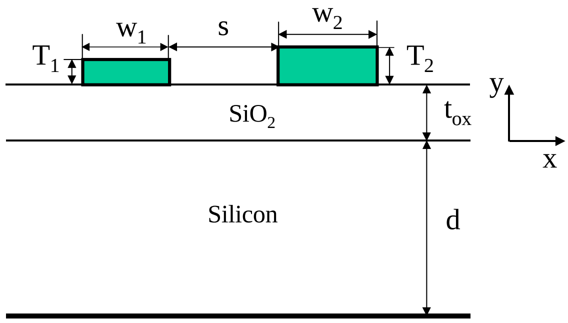
Simple and accurate closed-form expressions for the frequency-dependent mutual inductance and resistance per unit length of coupled interconnects on lossy silicon substrate are presented. Based on the closed-form evaluation of the integral equations for the magnetic vector potential (taking into account the induced current density distribution inside silicon substrate), the closed-form expressions for the series impedance parameters are obtained. The validity of the proposed analytic solutions has been checked by a comparison with a quasi-TEM spectral domain approach and an equivalent-circuit modeling procedure.

1. INTRODUCTION

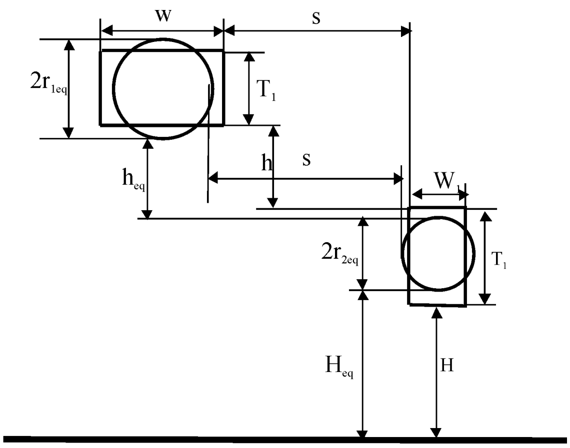
High frequency integrated circuits in CMOS (high loss substrates) technology are crucial components of today's integrated systems. As the density, complexity, and speed of IC circuits are continuing to increase, the management of the on-chip interconnects becomes of paramount concern to the IC designer, especially with respect to the internal parasitic parameters immunity [1]. In order to accomplish this, it is necessary to analyze and model the broad-band characteristics [2 - 5], [8], [9] of the silicon IC interconnects since the signals tend to exhibit both short rising and falling times. For the case of silicon the effect of high loss substrates (CMOS technology) on the distributed mutual inductance and resistance per unit length of coupled interconnects has not been modeled well with analytical closed form expressions. In this letter (based on silicon substrate induced current distribution) we suggest an analytical model that can accurately predict frequency dependent mutual inductance and resistance of silicon substrate IC interconnects, with good agreement with the quasi-TEM spectral domain approach and full wave numerical simulation [8], over a wide range of dimensions, substrate conductivities, and frequencies.

2. MODELING APPROACH

In order to investigate the influence of the longitudinal current distribution in the silicon substrate on the mutual inductance and resistance per unit length of the general coupled interconnects, the structure depicted in Fig. 1[(a), (b)] has been analyzed. To model actual rectangular conductors, we define an equivalent diameter $2r_{ieq}$ ($i = 1,2$) as the mean of the diameter of the circles inscribed in the conductors ($2r_{ieq} = (w_i + T_i)/2$). The other geometrical dimensions H, h and s are consequently redefined as $H_{eq} = H + (T_2-w_2)/4$, $h_{eq} = h + (T_1-w_1)/4 + (T_2-w_2)/4$ and $s_{eq} = s + (w_1-T_1)/4 + (w_2-T_2)/4$ (see Fig. 1(b)).



(a)



(b)

Fig. 1 **(a)** Cross section of coupled interconnects on an oxide/semiconductor substrate and **(b)** Round-sectioned interconnect lines over infinite thick silicon substrate

Due to the impressed field that interconnect lines radiate in the presence of a lossy silicon substrate, an unknown current density $\mathbf{J}_s$ is induced in the substrate: because of the particular geometry, this current has only a component along the z-axis, which is a function of x and y, namely $\mathbf{J}_s = J(x,y)\mathbf{1}_z$. Results obtained from the full-wave analysis [2] have shown that the influence of the finite substrate thickness d can be neglected for practical dimensions ($d >> w_1, w_2, s, t_{ox}$). Therefore in the following analysis we have assumed the silicon substrate to be infinitely thick. In order to derive the expression for mutual impedance Z_m of coupled interconnects a straight current filament parallel to a silicon semi-space will be first analyzed (see Fig. 1(b)). In the two regions depicted in Fig. 1(b), the governing equations for the magnetic vector potential is

$$\nabla^2 A_i = j\omega\mu_i\sigma_i A_i \quad i = 1,2 \quad (\sigma_1 = 0) \tag{1}$$

A general solution of eq. (1) may be looked for in the form [6]

$$A_1(x,y) = \int_0^\infty [C_{11}(\lambda)e^{\lambda y} + C_{12}(\lambda)e^{-\lambda y}]\cos(\lambda x)d\lambda,$$
$$for\ y \geq 0 \tag{2a}$$

$$A_2(x,y) = \int_0^\infty [C_2(\lambda)e^{my}\cos(\lambda x)d\lambda,$$
$$for\quad y \leq 0 \tag{2b}$$

where $m = (\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon))^{1/2}$.

The integration coefficients may be determined by imposing, at the interface silicon-silicon oxide (y = 0), the continuity of the tangential components of the magnetic field and of the normal component of the magnetic flux density. The following expressions are then obtained:

$$A_1(x,y) = \frac{\mu_0 I}{2\pi}\ln\frac{\sqrt{x^2+(y-b)^2}}{\sqrt{x^2+(y+b)^2}} +$$
$$+\frac{\mu I}{\pi}\int_0^\infty \frac{e^{-(b+y)\lambda}}{\mu_r\lambda + \sqrt{\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon)}}\cos(\lambda x)d\lambda \tag{3a}$$

$$A_2(x,y) =$$
$$= \frac{\mu I}{\pi}\int_0^\infty \frac{e^{-b\lambda}e^{y\sqrt{\lambda^2+j\omega\mu(\sigma+j\omega\varepsilon)}}}{\mu_r\lambda + \sqrt{\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon)}}\cos(\lambda x)d\lambda. \tag{3b}$$

In above expressions the magnetic potential is introduced in Maxwell's equations in order to find the current density distribution in the silicon substrate as $J_s(x,y) = -j\omega\sigma A_2(x,y)$. This leads to the quasi-static voltage drop $\partial V/\partial z$ in the z-direction that also appears in the classical transmission line equations,

$$\frac{\partial V}{\partial z} = -[Z]I\ . \tag{4}$$

The magnetic vector potential is used in order to find the quasi-static potential drop $\partial V/\partial z$ at any point (x,y) in the space along the lines parallel to the z direction. This allows the mutual impedance per unit length, eq. (4), to be evaluated.

The axial electric field intensity along the lossy silicon substrate is

$$E_{zs}(x, y=0) = -j\omega A_1(x, y=0) - \frac{\partial V(x, y=0)}{\partial z}, \tag{5}$$

and at any point (x,y) above the silicon substrate

$$E_{za}(x,y) = -j\omega A_1(x,y) - \frac{\partial V(x,y)}{\partial z}, \tag{6}$$

Substracting eq. (5) from eq. (6), the axial electric field intensity at any point above the lossy sil0,icon substrate can be expressed as

$$E_{za}(x,y) = E_{zs}(x,y=0) - j\omega[A_1(x,y) - A_1(x,y=0)]$$
$$-\frac{\partial}{\partial z}[V(x,y) - V(x,y=0)]\,. \tag{7}$$

The last term in eq. (7) represents the total scalar voltage drop, in the axial z-direction, of the distributed parameter circuit consisting of interconnect conductors and silicon substrate (as return), eq. (4).

In the expression for mutual impedance per unit length of coupled interconnects the integral parts contain the term of the form $(\sqrt{\lambda^2+\gamma^2} - \lambda)/(\sqrt{\lambda^2+\gamma^2} + \lambda)$, where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$. Introducing the following approximation for this term [7]

$$\frac{\sqrt{\lambda^2+\gamma^2} - \lambda}{\sqrt{\lambda^2+\gamma^2} + \lambda} \approx e^{-2\frac{\lambda}{\gamma}}\left[1 + \frac{\lambda^3}{3\gamma^3} - \frac{3\lambda^5}{20\gamma^5} + \ldots\right] \tag{8}$$

the closed form evaluation of these integrals can be done (the integration in the complex plane) [7] and the following closed-form formula for mutual impedance is obtained:

$$Z_m = \frac{j\omega\mu_0\mu_r}{2\pi}\left[\ln\left(\sqrt{\frac{D_{kn}^2 + \left(h_k + h_n + \frac{2}{jk_s}\right)^2}{D_{kn}^2 + (h_k - h_n)^2}}\right) - \right.$$

$$\left. - \frac{2\left(h_k + h_n + \dfrac{2}{jk_s}\right)}{3(jk_s)^3} \frac{\left(h_k + h_n + \dfrac{2}{jk_s}\right)^2 - 3D_{kn}^2}{\left[\left(h_k + h_n + \dfrac{2}{jk_s}\right)^2 + D_{kn}^2\right]^3} \right] \quad (9)$$

where $k_s = \sqrt{-j\omega\mu(\sigma + j\omega\varepsilon)}$, being j the imaginary unit, $h_k = H_{eq} + h_{eq} + 2r_{2eq} + r_{1eq}$, $h_n = H_{eq} + r_{2eq}$ and $D_{kn} = s_{eq} + r_{1eq} + r_{2eq}$, respectively.

3. RESULTS

In order to validate the derived closed-form formulas for series mutual impedance per unit length ($Z_m = R_m + j\omega L_m$), an asymmetric coupled interconnect structure on a 300 μm silicon substrate (resistivity ρ_{si} = 0.01 Ωcm) with a 3 μm oxide layer is considered. The cross sections of the conductors are 2 μm by 1 μm and 1 μm by 1 μm, respectively. The spacing between the two conductors is 2 μm. Fig. 2 shows the variation in the distributed mutual resistance per unit length, $R_m(\omega)$, as a function of a frequency. At higher frequencies, the increase of mutual resistance is enormous. The cause for this phenomenon can be found in the generation of the eddy currents in the silicon substrate.

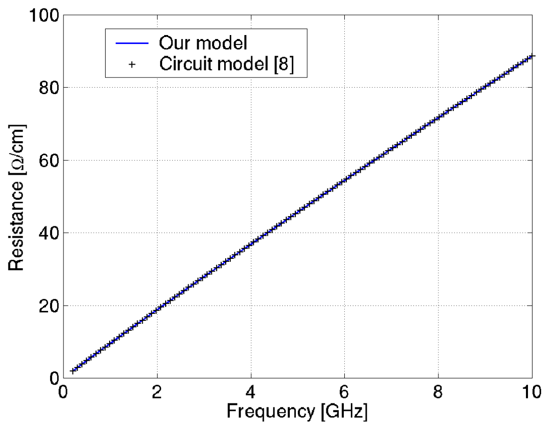


Fig. 2 Mutual resistance per unit length R_m as a function of frequency

Similarly, Fig. 3 shows the variation of the distributed mutual inductance per unit length, $L_m(\omega)$, as the function of a frequency. When the substrate conductivity is high, a skin-effect arises in substrate and the return currents flows more in the silicon substrate. The variation of the mutual inductance per unit length decreases rapidly as a function frequency in Fig. 3 since most of the induced current is confined in a limited zone of the silicon substrate just beneath the source lines (the skin-effect mode). At low frequencies the mutual inductance is high due to the slow-wave mode. It is observed that the values of the mutual inductance and resistance per unit length, calculated from derived formulas, are found to be in good agreement with those of [8] (equivalent-circuit models and quasi-TEM approach).

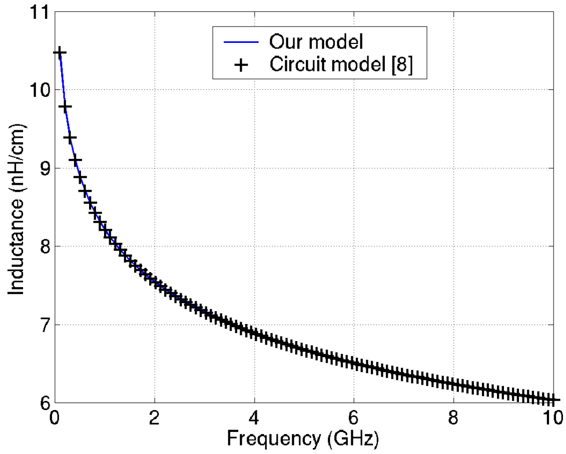


Fig. 3 Mutual inductance per unit length L_m as a function of frequency

4. CONCLUSION

In this paper we have developed a simple, highly accurate and low time consuming analytical formulas for the frequency-dependent mutual inductance and resistance per unit length of general multiple coupled IC interconnects on lossy silicon (CMOS) substrate for CAD usage, provided that no important skin effect occurs in the strip conductors with finite thickness. It was shown that the frequency-dependent mutual inductance and resistance per unit length can be accurately obtained in terms of closed-form solutions for integral equations of magnetic vector potential. Very good agreement of the calculation results from formulae with the field solver simulation has been achieved.

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BIOGRAPHY

Hasan Ymeri was born in Druar near Mitrovicë, Kosovë on 24 October 1957. He received the Dipl. Ing. (M. Sc.) degree in electronic engineering from University of Prishtina, Prishtinë, Kosovë, in 1983, the Mag. Sci. degree in electrical engineering from the University of Ljubljana, Slovenia, in 1988, and the Dr. Sc. degree in electronic engineering from the Polytechnic University, Tirana, Albania, in 1996, respectively.
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